ELECTRONIC RESEARCH ARCHIVE doi:10.3934/era.2021068

THE ALGEBRAIC CLASSIFICATION OF NILPOTENT COMMUTATIVE ALGEBRAS

Doston Jumaniyozov

National University of Uzbekistan, Tashkent, Uzbekistan
Institute of Mathematics named after V.I. Romanovsky Academy of Sciences of Uzbekistan
Tashkent, Uzbekistan

Ivan Kaygorodov*

CMCC, Universidade Federal do ABC, Santo André, Brazil
Moscow Center for Fundamental and Applied Mathematics, Moscow, Russia
Saint Petersburg University, Russia

Abror Khudoyberdiyev

National University of Uzbekistan, Tashkent, Uzbekistan
Institute of Mathematics named after V.I. Romanovsky
Academy of Sciences of Uzbekistan, Tashkent, Uzbekistan

Abstract. This paper is devoted to the complete algebraic classification of complex 5-dimensional nilpotent commutative algebras. Our method of classification is based on the standard method of classification of central extensions of smaller nilpotent commutative algebras and the recently obtained classification of complex 5-dimensional nilpotent commutative $\mathfrak{CD}$-algebras.

Introduction. The algebraic classification (up to isomorphism) of algebras of dimension n from a certain variety defined by a certain family of polynomial identities is a classic problem in the theory of non-associative algebras. There are many results related to the algebraic classification of small-dimensional algebras in many varieties of non-associative algebras [11, 12, 2, 3, 4, 6, 13, 9, 16]. So, algebraic classifications of 2-dimensional algebras [16, 19], 3-dimensional evolution algebras [1], 3-dimensional anticommutative algebras [17], 4-dimensional division algebras [7, 5], 4-dimensional nilpotent algebras [13] and 6-dimensional anticommutative nilpotent algebras [12] have been given. In the present paper, we give the algebraic classification of 5-dimensional nilpotent commutative algebras. The variety of commutative algebras is defined by the following identity: $xy = yx$. It contains commutative $\mathfrak{CD}$-algebras, Jordan algebras, mock-Lie algebras and commutative associative algebras as subvarieties. On the other hand, it is a principal part in the varieties of weakly associative algebras and flexible algebras.

The algebraic study of central extensions of associative and non-associative algebras has been an important topic for years (see, for example, [10, 20] and references therein). Our method for classifying nilpotent commutative algebras is based on the

2020 *Mathematics Subject Classification.* Primary: 17A01; Secondary: 17D99, 17A99.

Key words and phrases. Commutative algebras, Jordan algebras, nilpotent algebras, algebraic classification, central extension.

The work is supported by the Russian Science Foundation under grant 19-71-10016.

* Corresponding author: Ivan Kaygorodov.

calculation of central extensions of nilpotent algebras of smaller dimensions from the same variety (first, this method has been developed by Skjelbred and Sund for Lie algebra case in [20]) and the classifications of all complex 5-dimensional nilpotent commutative (non-Jordan) $\mathfrak{CD}$-algebras [11]; nilpotent Jordan (non-associative) algebras [9]; and nilpotent associative commutative algebras [18].

1. The algebraic classification of nilpotent commutative algebras.

1.1. Method of classification of nilpotent algebras. Throughout this paper, we use the notations and methods well written in [10], which we have adapted for the commutative case with some modifications. Further in this section we give some important definitions.

Let $(\mathbf{A},\cdot)$ be a complex commutative algebra and $\mathbb{V}$ be a complex vector space. The $\mathbb{C}$-linear space $\mathrm{Z}^2(\mathbf{A},\mathbb{V})$ is defined as the set of all bilinear maps $\theta\colon \mathbf{A}\times\mathbf{A}\longrightarrow\mathbb{V}$ such that $\theta(x,y)=\theta(y,x)$. These elements will be called *cocycles*. For a linear map f from $\mathbf{A}$ to $\mathbb{V}$, if we define $\delta f\colon \mathbf{A}\times\mathbf{A}\longrightarrow\mathbb{V}$ by $\delta f(x,y)=f(xy)$, then $\delta f\in\mathrm{Z}^2(\mathbf{A},\mathbb{V})$. We define $\mathrm{B}^2(\mathbf{A},\mathbb{V})=\{\theta=\delta f\ : f\in\mathrm{Hom}(\mathbf{A},\mathbb{V})\}$. We define the *second cohomology space* $\mathrm{H}^2(\mathbf{A},\mathbb{V})$ as the quotient space $\mathrm{Z}^2(\mathbf{A},\mathbb{V})\big/\mathrm{B}^2(\mathbf{A},\mathbb{V})$.

Let $\mathrm{Aut}(\mathbf{A})$ be the automorphism group of $\mathbf{A}$ and let $\phi\in\mathrm{Aut}(\mathbf{A})$. For $\theta\in\mathrm{Z}^2(\mathbf{A},\mathbb{V})$ define the action of the group $\mathrm{Aut}(\mathbf{A})$ on $\mathrm{Z}^2(\mathbf{A},\mathbb{V})$ by

$$\phi\theta(x,y)=\theta(\phi(x),\phi(y)).$$

It is easy to verify that $\mathrm{B}^2(\mathbf{A},\mathbb{V})$ is invariant under the action of $\mathrm{Aut}(\mathbf{A})$. So, we have an induced action of $\mathrm{Aut}(\mathbf{A})$ on $\mathrm{H}^2(\mathbf{A},\mathbb{V})$.

Let $\mathbf{A}$ be a commutative algebra of dimension m over $\mathbb{C}$ and $\mathbb{V}$ be a $\mathbb{C}$-vector space of dimension k. For the bilinear map θ, define on the linear space $\mathbf{A}_\theta=\mathbf{A}\oplus\mathbb{V}$ the bilinear product " $[-,-]_{\mathbf{A}_\theta}$" by $[x+x',y+y']_{\mathbf{A}_\theta}=xy+\theta(x,y)$ for all $x,y\in\mathbf{A},x',y'\in\mathbb{V}$. The algebra $\mathbf{A}_\theta$ is called a *k-dimensional central extension* of $\mathbf{A}$ by $\mathbb{V}$. One can easily check that $\mathbf{A}_\theta$ is a commutative algebra if and only if $\theta\in\mathrm{Z}^2(\mathbf{A},\mathbb{V})$.

Call the set $\mathrm{Ann}(\theta)=\{x\in\mathbf{A}:\theta(x,\mathbf{A})=0\}$ the *annihilator* of θ. We recall that the *annihilator* of an algebra $\mathbf{A}$ is defined as the ideal $\mathrm{Ann}(\mathbf{A})=\{x\in\mathbf{A}:x\mathbf{A}=0\}$. Observe that $\mathrm{Ann}(\mathbf{A}_\theta)=(\mathrm{Ann}(\theta)\cap\mathrm{Ann}(\mathbf{A}))\oplus\mathbb{V}$.

The following result shows that every algebra with a non-zero annihilator is a central extension of a smaller-dimensional algebra.

Lemma 1.1. *Let $\mathbf{A}$ be an n-dimensional commutative algebra such that*

$$\dim(\mathrm{Ann}(\mathbf{A}))=m\neq 0.$$

Then there exists, up to isomorphism, a unique $(n-m)$-dimensional commutative algebra $\mathbf{A}'$ and a bilinear map $\theta\in\mathrm{Z}^2(\mathbf{A}',\mathbb{V})$ with $\mathrm{Ann}(\mathbf{A}')\cap\mathrm{Ann}(\theta)=0$, where $\mathbb{V}$ is a vector space of dimension m, such that $\mathbf{A}\cong{\mathbf{A}'}_\theta$ and $\mathbf{A}/\mathrm{Ann}(\mathbf{A})\cong\mathbf{A}'$.

Proof. Let $\mathbf{A}'$ be a linear complement of $\mathrm{Ann}(\mathbf{A})$ in $\mathbf{A}$. Define a linear map $P\colon\mathbf{A}\longrightarrow\mathbf{A}'$ by $P(x+v)=x$ for $x\in\mathbf{A}'$ and $v\in\mathrm{Ann}(\mathbf{A})$, and define a multiplication on $\mathbf{A}'$ by $[x,y]_{\mathbf{A}'}=P(xy)$ for $x,y\in\mathbf{A}'$. For $x,y\in\mathbf{A}$, we have

$$P(xy)=P((x-P(x)+P(x))(y-P(y)+P(y)))=P(P(x)P(y))=[P(x),P(y)]_{\mathbf{A}'}.$$

Since P is a homomorphism, $P(\mathbf{A})=\mathbf{A}'$ and $\mathbf{A}'$ is a commutative algebra and also $\mathbf{A}/\mathrm{Ann}(\mathbf{A})\cong\mathbf{A}'$, which gives us the uniqueness. Now, define the map $\theta\colon\mathbf{A}'\times\mathbf{A}'\longrightarrow\mathrm{Ann}(\mathbf{A})$ by $\theta(x,y)=xy-[x,y]_{\mathbf{A}'}$. Thus, $\mathbf{A}'_\theta$ is $\mathbf{A}$ and therefore $\theta\in\mathrm{Z}^2(\mathbf{A}',\mathbb{V})$ and $\mathrm{Ann}(\mathbf{A}')\cap\mathrm{Ann}(\theta)=0$. □

Definition 1.2. Let $\mathbf{A}$ be an algebra and I be a subspace of $\mathrm{Ann}(\mathbf{A})$. If $\mathbf{A}=\mathbf{A}_0\oplus I$ then I is called an *annihilator component* of $\mathbf{A}$. A central extension of an algebra $\mathbf{A}$ without annihilator component is called a *non-split central extension*.

Our task is to find all central extensions of an algebra $\mathbf{A}$ by a space $\mathbb{V}$. In order to solve the isomorphism problem we need to study the action of $\mathrm{Aut}(\mathbf{A})$ on $\mathrm{H}^2(\mathbf{A},\mathbb{V})$. To do that, let us fix a basis $e_1,\ldots,e_s$ of $\mathbb{V}$, and $\theta\in\mathrm{Z}^2(\mathbf{A},\mathbb{V})$. Then θ can be uniquely written as $\theta(x,y)=\sum\limits_{i=1}^{s}\theta_i(x,y)e_i$, where $\theta_i\in\mathrm{Z}^2(\mathbf{A},\mathbb{C})$. Moreover, $\mathrm{Ann}(\theta)=\mathrm{Ann}(\theta_1)\cap\mathrm{Ann}(\theta_2)\cap\ldots\cap\mathrm{Ann}(\theta_s)$. Furthermore, $\theta\in\mathrm{B}^2(\mathbf{A},\mathbb{V})$ if and only if all $\theta_i\in\mathrm{B}^2(\mathbf{A},\mathbb{C})$. It is not difficult to prove (see [10, Lemma 13]) that given a commutative algebra $\mathbf{A}_\theta$, if we write as above $\theta(x,y)=\sum\limits_{i=1}^{s}\theta_i(x,y)e_i\in\mathrm{Z}^2(\mathbf{A},\mathbb{V})$ and $\mathrm{Ann}(\theta)\cap\mathrm{Ann}(\mathbf{A})=0$, then $\mathbf{A}_\theta$ has an annihilator component if and only if $[\theta_1],[\theta_2],\ldots,[\theta_s]$ are linearly dependent in $\mathrm{H}^2(\mathbf{A},\mathbb{C})$.

Let $\mathbb{V}$ be a finite-dimensional vector space over $\mathbb{C}$. The *Grassmannian* $G_k(\mathbb{V})$ is the set of all k-dimensional linear subspaces of $\mathbb{V}$. Let $G_s\left(\mathrm{H}^2(\mathbf{A},\mathbb{C})\right)$ be the Grassmannian of subspaces of dimension s in $\mathrm{H}^2(\mathbf{A},\mathbb{C})$. There is a natural action of $\mathrm{Aut}(\mathbf{A})$ on $G_s\left(\mathrm{H}^2(\mathbf{A},\mathbb{C})\right)$. Let $\phi\in\mathrm{Aut}(\mathbf{A})$. For $W=\langle[\theta_1],[\theta_2],\ldots,[\theta_s]\rangle\in G_s\left(\mathrm{H}^2(\mathbf{A},\mathbb{C})\right)$ define $\phi W=\langle[\phi\theta_1],[\phi\theta_2],\ldots,[\phi\theta_s]\rangle$. We denote the orbit of $W\in G_s\left(\mathrm{H}^2(\mathbf{A},\mathbb{C})\right)$ under the action of $\mathrm{Aut}(\mathbf{A})$ by $\mathrm{Orb}(W)$. Given

$$W_1=\langle[\theta_1],[\theta_2],\ldots,[\theta_s]\rangle, W_2=\langle[\vartheta_1],[\vartheta_2],\ldots,[\vartheta_s]\rangle\in G_s\left(\mathrm{H}^2(\mathbf{A},\mathbb{C})\right),$$

we easily have that if $W_1=W_2$, then $\bigcap\limits_{i=1}^{s}\mathrm{Ann}(\theta_i)\cap\mathrm{Ann}(\mathbf{A})=\bigcap\limits_{i=1}^{s}\mathrm{Ann}(\vartheta_i)\cap\mathrm{Ann}(\mathbf{A})$, and therefore we can introduce the set

$$\mathbf{T}_s(\mathbf{A})=\left\{W=\langle[\theta_1],\ldots,[\theta_s]\rangle\in G_s\left(\mathrm{H}^2(\mathbf{A},\mathbb{C})\right):\bigcap_{i=1}^{s}\mathrm{Ann}(\theta_i)\cap\mathrm{Ann}(\mathbf{A})=0\right\},$$

which is stable under the action of $\mathrm{Aut}(\mathbf{A})$.

Now, let $\mathbb{V}$ be an s-dimensional linear space and let us denote by $\mathbf{E}(\mathbf{A},\mathbb{V})$ the set of all *non-split s-dimensional central extensions* of $\mathbf{A}$ by $\mathbb{V}$. By above, we can write

$$\mathbf{E}(\mathbf{A},\mathbb{V})=\left\{\mathbf{A}_\theta:\theta(x,y)=\sum_{i=1}^{s}\theta_i(x,y)e_i \text{ and } \langle[\theta_1],[\theta_2],\ldots,[\theta_s]\rangle\in\mathbf{T}_s(\mathbf{A})\right\}.$$

We also have the following result, which can be proved as in [10, Lemma 17].

Lemma 1.3. *Let $\mathbf{A}_\theta,\mathbf{A}_\vartheta\in\mathbf{E}(\mathbf{A},\mathbb{V})$. Suppose that $\theta(x,y)=\sum\limits_{i=1}^{s}\theta_i(x,y)e_i$ and $\vartheta(x,y)=\sum\limits_{i=1}^{s}\vartheta_i(x,y)e_i$. Then the commutative algebras $\mathbf{A}_\theta$ and $\mathbf{A}_\vartheta$ are isomorphic if and only if*

$$\mathrm{Orb}\langle[\theta_1],[\theta_2],\ldots,[\theta_s]\rangle=\mathrm{Orb}\langle[\vartheta_1],[\vartheta_2],\ldots,[\vartheta_s]\rangle.$$

This shows that there exists a one-to-one correspondence between the set of $\mathrm{Aut}(\mathbf{A})$-orbits on $\mathbf{T}_s(\mathbf{A})$ and the set of isomorphism classes of $\mathbf{E}(\mathbf{A},\mathbb{V})$. Consequently we have a procedure that allows us, given a commutative algebra $\mathbf{A}'$ of dimension $n-s$, to construct all non-split central extensions of $\mathbf{A}'$. This procedure is:

1. For a given commutative algebra $\mathbf{A}'$ of dimension $n-s$, determine $\mathrm{H}^2(\mathbf{A}',\mathbb{C})$, $\mathrm{Ann}(\mathbf{A}')$ and $\mathrm{Aut}(\mathbf{A}')$.
2. Determine the set of $\mathrm{Aut}(\mathbf{A}')$-orbits on $\mathbf{T}_s(\mathbf{A}')$.
3. For each orbit, construct the commutative algebra associated with a representative of it.

1.2. **Reduction to non-$\mathfrak{CCD}$-algebras.** The idea of the definition of a $\mathfrak{CD}$-algebra comes from the following property of Jordan and Lie algebras: *the commutator of any pair of multiplication operators is a derivation*. This gives three identities of degree four, which reduce to only one identity of degree four in the commutative or anticommutative case. Namely, a commutative algebra is a commutative $\mathfrak{CD}$-algebra ($\mathfrak{CCD}$-algebra) if it satisfies the following identity:

$$((xy)a)b+((xb)a)y+x((yb)a)=((xy)b)a+((xa)b)y+x((ya)b).$$

The above described method gives all commutative ($\mathfrak{CCD}$- and non-$\mathfrak{CCD}$-) algebras. But we are interested in developing this method in such a way that it only gives non-$\mathfrak{CCD}$ commutative algebras, because the classification of all $\mathfrak{CCD}$-algebras is done in [11]. Clearly, any central extension of a commutative non-$\mathfrak{CCD}$-algebra is a non-$\mathfrak{CCD}$-algebra. But a $\mathfrak{CCD}$-algebra may have extensions which are not $\mathfrak{CCD}$-algebras. More precisely, let $\mathfrak{D}$ be a $\mathfrak{CCD}$-algebra and $\theta\in \mathrm{Z}^2_{\mathfrak{C}}(\mathfrak{D},\mathbb{C})$. Then $\mathfrak{D}_\theta$ is a $\mathfrak{CCD}$-algebra if and only if

$$\theta(x,y)=\theta(y,x),$$
$$\theta((xy)a,b)+\theta((xb)a,y)+\theta(x,(yb)a)=\theta((xy)b,a)+\theta((xa)b,y)+\theta(x,(ya)b).$$

for all $x,y,a,b\in\mathfrak{D}$. Define the subspace $\mathrm{Z}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C})$ of $\mathrm{Z}^2_{\mathfrak{C}}(\mathfrak{D},\mathbb{C})$ by

$$\mathrm{Z}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C})=\left\{\begin{array}{c}\theta\in \mathrm{Z}^2_{\mathfrak{C}}(\mathfrak{D},\mathbb{C}):\theta(x,y)=\theta(y,x),\\ \theta((xy)a,b)+\theta((xb)a,y)+\theta(x,(yb)a)=\\ \theta((xy)b,a)+\theta((xa)b,y)+\theta(x,(ya)b)\\ \text{for all } x,y,a,b\in\mathfrak{D}\end{array}\right\}.$$

Observe that $\mathrm{B}^2(\mathfrak{D},\mathbb{C})\subseteq \mathrm{Z}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C})$. Let $\mathrm{H}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C})=\mathrm{Z}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C})\big/\mathrm{B}^2(\mathfrak{D},\mathbb{C})$. Then $\mathrm{H}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C})$ is a subspace of $\mathrm{H}^2_{\mathfrak{C}}(\mathfrak{D},\mathbb{C})$. Define

$$\mathbf{R}_s(\mathfrak{D})=\left\{\mathbf{W}\in\mathbf{T}_s(\mathfrak{D}):\mathbf{W}\in G_s(\mathrm{H}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C}))\right\},$$
$$\mathbf{U}_s(\mathfrak{D})=\left\{\mathbf{W}\in\mathbf{T}_s(\mathfrak{D}):\mathbf{W}\notin G_s(\mathrm{H}^2_{\mathfrak{D}}(\mathfrak{D},\mathbb{C}))\right\}.$$

Then $\mathbf{T}_s(\mathfrak{D})=\mathbf{R}_s(\mathfrak{D})\mathbin{\dot\cup}\mathbf{U}_s(\mathfrak{D})$. The sets $\mathbf{R}_s(\mathfrak{D})$ and $\mathbf{U}_s(\mathfrak{D})$ are stable under the action of $\mathrm{Aut}(\mathfrak{D})$. Thus, the commutative algebras corresponding to the representatives of $\mathrm{Aut}(\mathfrak{D})$-orbits on $\mathbf{R}_s(\mathfrak{D})$ are $\mathfrak{CCD}$-algebras, while those corresponding to the representatives of $\mathrm{Aut}(\mathfrak{D})$-orbits on $\mathbf{U}_s(\mathfrak{D})$ are not $\mathfrak{CCD}$-algebras. Hence, we may construct all non-split commutative non-$\mathfrak{CCD}$-algebras $\mathbf{A}$ of dimension n with s-dimensional annihilator from a given commutative algebra $\mathbf{A}'$ of dimension $n-s$ in the following way:

1. If $\mathbf{A}'$ is non-$\mathfrak{CCD}$, then apply the procedure.
2. Otherwise, do the following:
   (a) Determine $\mathbf{U}_s(\mathbf{A}')$ and $\mathrm{Aut}(\mathbf{A}')$.
   (b) Determine the set of $\mathrm{Aut}(\mathbf{A}')$-orbits on $\mathbf{U}_s(\mathbf{A}')$.
   (c) For each orbit, construct the commutative algebra corresponding to one of its representatives.

1.3. **Notations.** Let us introduce the following notations. Let $\mathbf{A}$ be a nilpotent algebra with a basis $e_1, e_2, \ldots, e_n$. Then by Δ_{ij} we will denote the bilinear form $\Delta_{ij} : \mathbf{A} \times \mathbf{A} \longrightarrow \mathbb{C}$ with $\Delta_{ij}(e_l, e_m) = \delta_{il}\delta_{jm}$, if $i \leq j$ and $l \leq m$. The set $\{\Delta_{ij} : 1 \leq i \leq j \leq n\}$ is a basis for the linear space of bilinear forms on $\mathbf{A}$, so every $\theta \in \mathrm{Z}^2(\mathbf{A}, \mathbb{V})$ can be uniquely written as $\theta = \sum\limits_{1 \leq i \leq j \leq n} c_{ij}\Delta_{ij}$, where $c_{ij} \in \mathbb{C}$. Let us fix complex number η_k $\ (\eta_k^k = -1,\ \eta_k^l \neq 1$ for $0 < l < k)$. For denote our algebras, we will use the following notations:

$\mathbf{N}_j^{\Xi}$	—	jth 5-dimensional family of commutative non-$\mathfrak{CCD}$-algebras with parametrs Ξ.
$\mathbf{N}_j^{i}$	—	jth i-dimensional non-$\mathfrak{CCD}$-algebra.
$\mathbf{N}_j^{i*}$	—	jth i-dimensional $\mathfrak{CCD}$-algebra.

Remark 1. All families of algebras from our final list do not have intersections, but inside some families of algebras there are isomorphic algebras. All isomorphisms between algebras from a certain family of algebras constucted from the representative $\nabla(\Sigma)$ are given in the list of distinct orbit representations. The notation $\langle\nabla(\Xi)\rangle^{O(\Xi_1)=O(\Xi_2)}$ represents that the elements $\langle\nabla(\Xi_1)\rangle$ and $\langle\nabla(\Xi_2)\rangle$ have the same orbit.

1.4. **3- and 4-dimensional commutative algebras.** Thanks to [8] we have the complete classification of complex 4-dimensional nilpotent commutative algebras. It will be re-written by some different way for separating $\mathfrak{CCD}$- and non-$\mathfrak{CCD}$-algebras.

$\mathbf{N}_{01}^{3*}, \mathbf{N}_{01}^{4*}$	:	$e_1e_1 = e_2$				$\mathrm{H}^2_{\mathfrak{C}} = \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{02}^{3*}, \mathbf{N}_{02}^{4*}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$			$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{03}^{3*}, \mathbf{N}_{03}^{4*}$	:	$e_1e_2 = e_3$				$\mathrm{H}^2_{\mathfrak{C}} = \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{04}^{3*}, \mathbf{N}_{04}^{4*}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$			$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{05}^{4*}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$			$\mathrm{H}^2_{\mathfrak{C}} = \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{06}^{4*}$	:	$e_1e_1 = e_2$	$e_3e_3 = e_4$			$\mathrm{H}^2_{\mathfrak{C}} = \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{07}^{4*}$	:	$e_1e_1 = e_4$	$e_2e_3 = e_4$			$\mathrm{H}^2_{\mathfrak{C}} = \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{08}^{4*}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_4$		$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{09}^{4*}$	:	$e_1e_1 = e_2$	$e_2e_3 = e_4$			$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{10}^{4*}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_3e_3 = e_4$		$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{11}^{4*}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$		$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{12}^{4*}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_4$	$e_3e_3 = e_4$		$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{13}^{4*}(\lambda)$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \lambda e_4$	$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{14}^{4*}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$			$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{15}^{4*}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$		$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{16}^{4*}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$		$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{17}^{4*}$	:	$e_1e_2 = e_3$	$e_3e_3 = e_4$			$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{18}^{4*}$	:	$e_1e_1 = e_4$	$e_1e_2 = e_3$	$e_3e_3 = e_4$		$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{19}^{4*}$	:	$e_1e_1 = e_4$	$e_1e_2 = e_3$	$e_2e_2 = e_4$	$e_3e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{C}} \neq \mathrm{H}^2_{\mathfrak{D}}$
$\mathbf{N}_{01}^{4}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_4$		
$\mathbf{N}_{02}^{4}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	
$\mathbf{N}_{03}^{4}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_3e_3 = e_4$		
$\mathbf{N}_{04}^{4}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_4$	$e_3e_3 = e_4$	
$\mathbf{N}_{05}^{4}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$		
$\mathbf{N}_{06}^{4}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	

$\mathbf{N}^4_{07}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$		
$\mathbf{N}^4_{08}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	
$\mathbf{N}^4_{09}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_3e_3 = e_4$		
$\mathbf{N}^4_{10}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_1e_2 = e_4$	$e_3e_3 = e_4$	
$\mathbf{N}^4_{11}(\lambda)$	:	$e_1e_1 = e_2$	$e_1e_2 = \lambda e_4$	$e_2e_2 = e_3$		
		$e_2e_3 = e_4$	$e_3e_3 = e_4$			

2. Central extensions of 3-dimensional nilpotent commutative algebras.

2.1. 2-dimensional central extension of $\mathbf{N}^{3*}_{02}$. Here we will collect all information about $\mathbf{N}^{3*}_{02}$:

		Cohomology	Automorphisms
$\mathbf{N}^{3*}_{02}$	$e_1e_1 = e_2$ $e_1e_2 = e_3$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}^{3*}_{02}) = \langle [\Delta_{13}], [\Delta_{22}] \rangle,$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^{3*}_{02}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}^{3*}_{02}) \oplus \langle [\Delta_{23}], [\Delta_{33}] \rangle$	$\phi = \begin{pmatrix} x & 0 & 0 \\ y & x^2 & 0 \\ z & 2xy & x^3 \end{pmatrix}$

Let us use the following notations:

$$\nabla_1 = [\Delta_{13}], \quad \nabla_2 = [\Delta_{22}], \quad \nabla_3 = [\Delta_{23}], \quad \nabla_4 = [\Delta_{33}].$$

Take $\theta = \sum\limits_{i=1}^{4} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^{3*}_{02})$. Since

$$\phi^T \begin{pmatrix} 0 & 0 & \alpha_1 \\ 0 & \alpha_2 & \alpha_3 \\ \alpha_1 & \alpha_3 & \alpha_4 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha^{**} & \alpha_1^* \\ \alpha^{**} & \alpha_2^* & \alpha_3^* \\ \alpha_1^* & \alpha_3^* & \alpha_4^* \end{pmatrix},$$

we have

$$\begin{array}{ll} \alpha_1^* = (\alpha_1 x + \alpha_3 y + \alpha_4 z)x^3, & \alpha_2^* = (\alpha_2 x^2 + 4\alpha_3 xy + 4\alpha_4 y^2)x^2, \\ \alpha_3^* = (\alpha_3 x + 2\alpha_4 y)x^4, & \alpha_4^* = \alpha_4 x^6. \end{array}$$

We are interested only in $(\alpha_3, \alpha_4) \neq (0, 0)$ and consider the vector space generated by the following two cocycles:

$$\theta_1 = \alpha_1\nabla_1 + \alpha_2\nabla_2 + \alpha_3\nabla_3 + \alpha_4\nabla_4 \ \text{ and } \ \theta_2 = \beta_1\nabla_1 + \beta_2\nabla_2 + \beta_3\nabla_3.$$

Thus, we have

$$\begin{array}{ll} \alpha_1^* = (\alpha_1 x + \alpha_3 y + \alpha_4 z)x^3, & \beta_1^* = (\beta_1 x + \beta_3 y)x^3, \\ \alpha_2^* = (\alpha_2 x^2 + 4\alpha_3 xy + 4\alpha_4 y^2)x^2, & \beta_2^* = (\beta_2 x + 4\beta_3 y)x^3, \\ \alpha_3^* = (\alpha_3 x + 2\alpha_4 y)x^4, & \beta_3^* = \beta_3 x^5. \\ \alpha_4^* = \alpha_4 x^6. & \end{array}$$

Consider the following cases.

1. $\alpha_4 \neq 0$, then:
   (a) $\beta_3 = 0, \beta_2 \neq 0, \beta_1 = 0$, then by choosing $x = 2\alpha_4^2$, $y = -\alpha_3\alpha_4$, $z = \alpha_3^2 - 2\alpha_1\alpha_4$, we have the representatives $\langle \nabla_4, \nabla_2 \rangle$;
   (b) $\beta_3 = 0, \beta_2 \neq 0, \beta_1 \neq 0$, then by choosing
   $x = 2\alpha_4^2\beta_2$, $y = -\alpha_3\alpha_4\beta_2$, $z = \alpha_3^2(-2\beta_1 + \beta_2) + 2\alpha_4(\alpha_2\beta_1 - \alpha_1\beta_2)$,
   we have the representatives $\langle \nabla_4, \nabla_1 + \alpha\nabla_2 \rangle_{\alpha \neq 0}$;
   (c) $\beta_3 = 0, \beta_2 = 0, \beta_1 \neq 0$, then by choosing $y = -\frac{x\alpha_3}{2\alpha_4}$, we have two representatives $\langle \nabla_4, \nabla_1 \rangle$ and $\langle \nabla_2 + \nabla_4, \nabla_1 \rangle$, depending on $\alpha_3^2 = \alpha_2\alpha_4$ or not. The first representative will be joint with the family from the case (1b);

(d) $\beta_3 \neq 0, 4\alpha_2\beta_3^2 = 4\beta_2\alpha_3\beta_3 - \beta_2^2\alpha_4, \beta_2 = 4\beta_1$, then by choosing
$$x = 4\beta_3\alpha_4, y = -\beta_2\alpha_4, z = \beta_2\alpha_3 - 4\alpha_1\beta_3,$$
we have the representative $\langle\nabla_4, \nabla_3\rangle$;

(e) $\beta_3 \neq 0, 4\alpha_2\beta_3^2 = 4\beta_2\alpha_3\beta_3 - \beta_2^2\alpha_4, \beta_2 \neq 4\beta_1$, then by choosing
$$x = \frac{4\beta_1-\beta_2}{4\beta_3}, y = \frac{\beta_2^2-4\beta_1\beta_2}{16\beta_3^2}, z = \frac{(4\beta_1-\beta_2)(8\beta_1\alpha_3\beta_3-4\beta_1\beta_2\alpha_4-8\alpha_1\beta_3^3+\beta_2^2\alpha_4)}{32\beta_3^3\alpha_4},$$
we have the representative $\langle\nabla_4, \nabla_1 + \nabla_3\rangle$;

(f) $\beta_3 \neq 0, 4\alpha_2\beta_3^2 \neq 4\beta_2\alpha_3\beta_3 - \beta_2^2\alpha_4$, then by choosing
$$x = \sqrt{\frac{4\alpha_2\beta_3^2-4\beta_2\alpha_3\beta_3+\beta_2^2\alpha_4}{4\beta_3^2\alpha_4}}, y = -\frac{\beta_2\sqrt{\alpha_4\beta_2^2-4\alpha_3\beta_2\beta_3+4\alpha_2\beta_3^2}}{8\beta_3^2\sqrt{\alpha_4}},$$
$$z = \frac{(8\beta_1\alpha_3\beta_3-4\beta_1\beta_2\alpha_4-8\alpha_1\beta_3^3+\beta_2^2\alpha_4)\sqrt{4\alpha_2\beta_3^2-4\beta_2\alpha_3\beta_3+\beta_2^2\alpha_4}}{16\beta_3^3\alpha_4\sqrt{\alpha_4}},$$
we have the family of representatives $\langle\nabla_2 + \nabla_4, \alpha\nabla_1 + \nabla_3\rangle$.

2. $\alpha_4 = 0, \alpha_3 \neq 0$, then we may suppose that $\beta_3 = 0$ and
   (a) if $\beta_1 \neq 0, \beta_2 = 4\beta_1, \alpha_2 = 4\alpha_1$, then by choosing $x = \alpha_3, y = -\alpha_1, z = 0$, we have the representative $\langle\nabla_3, \nabla_1 + 4\nabla_2\rangle$;
   (b) if $\beta_1 \neq 0, \beta_2 = 4\beta_1, \alpha_2 \neq 4\alpha_1$, then by choosing $x = \frac{\alpha_2-4\alpha_1}{\alpha_3}, y = \frac{4\alpha_1^2-\alpha_1\alpha_2}{\alpha_3^2}, z = 0$, we have the representative $\langle -24(\nabla_2 + \nabla_3), \nabla_1 + 4\nabla_2\rangle$;
   (c) if $\beta_1 \neq 0, \beta_2 \neq 4\beta_1$, then by choosing $x = \alpha_3(\beta_2 - 4\beta_1), y = \beta_1\alpha_2 - \alpha_1\beta_2, z = 0$, we have the family of representatives $\langle\nabla_3, \nabla_1 + \alpha\nabla_2\rangle_{\alpha\neq4}$, which will be jointed with the case (2a);
   (d) if $\beta_1 = 0$, then we have the representative $\langle -3\nabla_3, \nabla_2\rangle$.

Summarizing, we have the following distinct orbits:
$$\langle\nabla_1, \nabla_2 + \nabla_4\rangle, \langle\nabla_1 + 4\nabla_2, -24(\nabla_2 + \nabla_3)\rangle, \langle\nabla_1 + \lambda\nabla_2, \nabla_3\rangle, \langle\nabla_1 + \lambda\nabla_2, \nabla_4\rangle,$$
$$\langle\alpha\nabla_1 + \nabla_3, \nabla_2 + \nabla_4\rangle, \langle\nabla_1 + \nabla_3, \nabla_4\rangle, \langle\nabla_2, -3\nabla_3\rangle, \langle\nabla_2, \nabla_4\rangle, \langle\nabla_3, \nabla_4\rangle.$$

Note that the algebras constructed from the orbits $\langle\nabla_1 + 4\nabla_2, -24(\nabla_2 + \nabla_3)\rangle$, $\langle\nabla_1 + \lambda\nabla_2, \nabla_3\rangle$, $\langle\nabla_1 + \alpha\nabla_2, \nabla_4\rangle$, $\langle\nabla_2, -3\nabla_3\rangle$ and $\langle\nabla_2, \nabla_4\rangle$ are parts of some families of algebras which found below. Hence, we have the following new algebras:

$\mathbf{N}_{12}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{168}^{4}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = 4e_4 - 24e_5$	$e_2e_3 = -24e_5$
$\mathbf{N}_{170}^{\lambda,0}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \lambda e_4$	$e_2e_3 = e_5$
$\mathbf{N}_{184}^{\lambda,0}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \lambda e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{13}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_4$		
		$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$		
$\mathbf{N}_{14}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{76}^{-1}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_4$	$e_2e_3 = -3e_5$	
$\mathbf{N}_{80}^{0}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_4$	$e_3e_3 = e_5$	
$\mathbf{N}_{15}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	

2.2. **2-dimensional central extension of $\mathbf{N}_{04}^{3*}$.** Here we will collect all information about $\mathbf{N}_{04}^{3*}$:

$\mathbf{N}_{04}^{3*}$	$e_1e_1 = e_2$ $e_2e_2 = e_3$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{04}^{3*}) = \langle[\Delta_{12}]\rangle$, $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{04}^{3*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{04}^{3*}) \oplus \langle[\Delta_{13}], [\Delta_{23}], [\Delta_{33}]\rangle$	$\phi = \begin{pmatrix} x & 0 & 0 \\ 0 & x^2 & 0 \\ z & 0 & x^4 \end{pmatrix}$

Let us use the following notations:
$$\nabla_1 = [\Delta_{12}], \quad \nabla_2 = [\Delta_{13}], \quad \nabla_3 = [\Delta_{23}], \quad \nabla_4 = [\Delta_{33}].$$

Take $\theta = \sum\limits_{i=1}^{4} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^{3*}_{04})$. Since

$$\phi^T \begin{pmatrix} 0 & \alpha_1 & \alpha_2 \\ \alpha_1 & 0 & \alpha_3 \\ \alpha_2 & \alpha_3 & \alpha_4 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha_1^* & \alpha_2^* \\ \alpha_1^* & \alpha^{**} & \alpha_3^* \\ \alpha_2^* & \alpha_3^* & \alpha_4^* \end{pmatrix},$$

we have

$$\alpha_1^* = (\alpha_1 x + \alpha_3 z)x^2, \quad \alpha_2^* = (\alpha_2 x + \alpha_4 z)x^4,$$
$$\alpha_3^* = \alpha_3 x^5, \quad \alpha_4^* = \alpha_4 x^8.$$

Consider the following cases:

1. $\alpha_4 \neq 0$, then consider the vector space generated by the following two cocycles:
$$\theta_1 = \alpha_1\nabla_1 + \alpha_2\nabla_2 + \alpha_3\nabla_3 + \alpha_4\nabla_4 \quad \text{and} \quad \theta_2 = \beta_1\nabla_1 + \beta_2\nabla_2 + \beta_3\nabla_3.$$
Thus, we have
$$\alpha_1^* = (\alpha_1 x + \alpha_3 z)x^2, \quad \beta_1^* = (\beta_1 x + \beta_3 z)x^2,$$
$$\alpha_2^* = (\alpha_2 x + \alpha_4 z)x^4, \quad \beta_2^* = \beta_2 x^5,$$
$$\alpha_3^* = \alpha_3 x^6, \quad \beta_3^* = \beta_3 x^6.$$
$$\alpha_4^* = \alpha_4 x^8.$$

Then we consider the following subcases:

(a) $\beta_3 = 0, \alpha_3 = 0$, then we have:
 (i) if $\beta_1 = 0, \alpha_1 = 0$, then we have the representative $\langle \nabla_4, \nabla_2 \rangle$;
 (ii) if $\beta_1 = 0, \alpha_1 \neq 0$, then by choosing $x = \sqrt[5]{\alpha_1 {\alpha_4}^{-1}}$, we have the representative $\langle \nabla_1 + \nabla_4, \nabla_2 \rangle$;
 (iii) if $\beta_1 \neq 0, \beta_2 = 0$, then by choosing $x = 1$ and $z = -\alpha_2\alpha_4^{-1}$, we have the representative $\langle \nabla_4, \nabla_1 \rangle$;
 (iv) if $\beta_1 \neq 0, \beta_2 \neq 0$, then by choosing $x = \sqrt{\beta_1 {\beta_2}^{-1}}$ and $z = \frac{\alpha_1\beta_2 - \beta_1\alpha_2}{\alpha_4\sqrt{\beta_1\beta_2}}$, we have the representative $\langle \nabla_4, \nabla_1 + \nabla_2 \rangle$.

(b) $\beta_3 = 0, \alpha_3 \neq 0$, then we have:
 (i) if $\beta_2 = 0$, then by choosing $x = \sqrt{\alpha_3 {\alpha_4}^{-1}}$ and $z = -\alpha_2\sqrt{\alpha_3\alpha_4^{-3}}$, we have the representative $\langle \nabla_3 + \nabla_4, \nabla_1 \rangle$;
 (ii) if $\beta_2 \neq 0, \beta_1 = 0$, then by choosing
$$x = \sqrt{\alpha_3 {\alpha_4}^{-1}} \text{ and } z = -\alpha_1\sqrt{{\alpha_3}^{-1}\alpha_4^{-1}},$$
 we have the representative $\langle \nabla_3 + \nabla_4, \nabla_2 \rangle$;
 (iii) if $\beta_2 \neq 0, \beta_1 \neq 0, \beta_2\alpha_3 = \beta_1\alpha_4$, then by choosing $x = \sqrt{\alpha_3 {\alpha_4}^{-1}}$ and $z = 0$, we have the family of representatives $\langle \alpha\nabla_1 + \nabla_3 + \nabla_4, \nabla_1 + \nabla_2 \rangle$;
 (iv) if $\beta_2 \neq 0, \beta_1 \neq 0, \beta_2\alpha_3 \neq \beta_1\alpha_4$, then by choosing
$$x = \sqrt{\tfrac{\beta_1}{\beta_2}} \text{ and } z = \tfrac{(\alpha_1\beta_2 - \beta_1\alpha_2)\sqrt{\beta_1}}{(\beta_1\alpha_4 - \beta_2\alpha_3)\sqrt{\beta_2}},$$
 we have the family of representatives $\langle \alpha\nabla_3 + \nabla_4, \nabla_1 + \nabla_2 \rangle_{\alpha \neq 0,1}$, which will be jointed with the case (1(a)iv).

(c) $\beta_3 \neq 0, \alpha_3 = 0$, then we have:
 (i) if $\beta_2 = 0, \alpha_1 \neq 0$, then by choosing $x = \sqrt[5]{\alpha_1\alpha_4^{-1}}$ and $z = -\alpha_2 \sqrt[5]{\alpha_1\alpha_4^{-6}}$, we have the family of representatives $\langle \nabla_1 + \nabla_4, \alpha\nabla_1 + \nabla_3 \rangle$;

(ii) if $\beta_2=0, \alpha_1=0$, then by choosing $z=-\frac{\alpha_2 x}{\alpha_4}$, we have two representatives $\langle \nabla_4, \nabla_3\rangle$ or $\langle \nabla_4, \nabla_1+\nabla_3\rangle$ depending on whether $\beta_1\alpha_4=\alpha_2\beta_3$ or not;

(iii) if $\beta_2\neq 0$, then by choosing $x=\frac{\beta_2}{\beta_3}$ and $z=-\frac{\alpha_2\beta_2}{\beta_3\alpha_4}$, we have the family of representatives $\langle \alpha\nabla_1+\nabla_4, \beta\nabla_1+\nabla_2+\nabla_3\rangle$.

2. $\alpha_4=0, \alpha_3\neq 0$, then we may suppose that $\beta_3=0$. Thus, we have

$$\begin{array}{ll} \alpha_1^*=(\alpha_1 x+\alpha_3 z)x^2, & \beta_1^*=\beta_1 x^3, \\ \alpha_2^*=\alpha_2 x^5, & \beta_2^*=\beta_2 x^5, \\ \alpha_3^*=\alpha_3 x^6, & \end{array}$$

and consider the following subcases:

(a) $\beta_2=0$, then we have two representatives $\langle \nabla_3, \nabla_1\rangle$ or $\langle \nabla_2+\nabla_3, \nabla_1\rangle$, depending on whether $\alpha_2=0$ or not;

(b) $\beta_2\neq 0, \alpha_2=0$, then by choosing $z=-\frac{\alpha_1 x}{\alpha_3}$, we have two representatives $\langle \nabla_3, \nabla_2\rangle$ or $\langle \nabla_3, \nabla_1+\nabla_2\rangle$, depending on whether $\beta_1=0$ or not.

3. $\alpha_4=0, \alpha_3=0, \beta_3=0, \beta_2=0, \alpha_2\neq 0$, then we have the representative $\langle \nabla_2, \nabla_1\rangle$.

Summarizing, we have the following distinct orbits:

$$\langle \nabla_1, \nabla_2\rangle, \langle \nabla_1, \nabla_2+\nabla_3\rangle, \langle \nabla_1, \nabla_3\rangle, \langle \nabla_1, \nabla_3+\nabla_4\rangle, \langle \nabla_1, \nabla_4\rangle,$$
$$\langle \nabla_1+\nabla_2, \alpha\nabla_1+\nabla_3+\nabla_4\rangle^{O(\alpha)=O(-\alpha)}, \langle \nabla_1+\nabla_2, \nabla_3\rangle, \langle \nabla_1+\nabla_2, \alpha\nabla_3+\nabla_4\rangle_{\alpha\neq 1},$$
$$\langle \beta\nabla_1+\nabla_2+\nabla_3, \alpha\nabla_1+\nabla_4\rangle, \langle \alpha\nabla_1+\nabla_3, \nabla_1+\nabla_4\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},$$
$$\langle \nabla_1+\nabla_3, \nabla_4\rangle, \langle \nabla_1+\nabla_4, \nabla_2\rangle, \langle \nabla_2, \nabla_3\rangle, \langle \nabla_2, \nabla_3+\nabla_4\rangle, \langle \nabla_2, \nabla_4\rangle, \langle \nabla_3, \nabla_4\rangle.$$

Note that, the orbit $\langle \nabla_1, \nabla_2\rangle$ after a change of the basis of the constructed algebra gives a part of the family $\mathbf{N}_{79}^{\alpha}$, which will be found below. Hence, we have the following new algebras:

$\mathbf{N}_{76}^{0}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=e_4$	
$\mathbf{N}_{16}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$
$\mathbf{N}_{17}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_2=e_3$	$e_2e_3=e_5$	
$\mathbf{N}_{18}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{19}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_2=e_3$	$e_3e_3=e_5$	
$\mathbf{N}_{20}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_4+\alpha e_5$	$e_1e_3=e_4$		
		$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{21}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_3=e_5$
$\mathbf{N}_{22}^{\alpha\neq 1}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_4$		
		$e_2e_2=e_3$	$e_2e_3=\alpha e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{23}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=\beta e_4+\alpha e_5$	$e_1e_3=e_4$		
		$e_2e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=e_5$		
$\mathbf{N}_{24}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=\alpha e_4+e_5$	$e_2e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=e_5$
$\mathbf{N}_{25}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=e_5$
$\mathbf{N}_{26}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_3e_3=e_4$
$\mathbf{N}_{27}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_3=e_5$	
$\mathbf{N}_{28}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{29}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_3e_3=e_5$	
$\mathbf{N}_{30}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=e_5$	

3. Central extensions of nilpotent $\mathfrak{CCD}$-algebras.

3.1. 1-dimensional central extensions of $\mathbf{N}_{02}^{4*}$. Here we will collect all information about $\mathbf{N}_{02}^{4*}$:

$\mathbf{N}_{02}^{4*}$	$e_1e_1 = e_2$ $e_1e_2 = e_3$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{02}^{4*}) = \langle [\Delta_{13}], [\Delta_{22}], [\Delta_{14}], [\Delta_{24}], [\Delta_{44}] \rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{02}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{02}^{4*}) \oplus \langle [\Delta_{23}], [\Delta_{33}], [\Delta_{34}] \rangle$	$\phi = \begin{pmatrix} x & 0 & 0 & 0 \\ q & x^2 & 0 & 0 \\ w & 2xq & x^3 & r \\ e & 0 & 0 & t \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{13}], & \nabla_2 = [\Delta_{14}], & \nabla_3 = [\Delta_{22}], & \nabla_4 = [\Delta_{23}], \\ \nabla_5 = [\Delta_{24}], & \nabla_6 = [\Delta_{33}], & \nabla_7 = [\Delta_{34}], & \nabla_8 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{02}^{4*})$. Since

$$\phi^T \begin{pmatrix} 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 & \alpha_5 \\ \alpha_1 & \alpha_4 & \alpha_6 & \alpha_7 \\ \alpha_2 & \alpha_5 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha^{**} & \alpha_1^* & \alpha_2^* \\ \alpha^{**} & \alpha_3^* & \alpha_4^* & \alpha_5^* \\ \alpha_1^* & \alpha_4^* & \alpha_6^* & \alpha_7^* \\ \alpha_2^* & \alpha_5^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

we have

$$\begin{array}{rcl} \alpha_1^* &=& (\alpha_1 x + \alpha_4 q + \alpha_6 w + \alpha_7 e)x^3, \\ \alpha_2^* &=& (\alpha_1 x + \alpha_4 q + \alpha_6 w + \alpha_7 e)r + (\alpha_2 x + \alpha_5 q + \alpha_7 w + \alpha_8 e)t, \\ \alpha_3^* &=& (\alpha_3 x^2 + 4\alpha_4 xq + 4\alpha_6 q^2)x^2, \\ \alpha_4^* &=& (\alpha_4 x + 2\alpha_6 q)x^4, \\ \alpha_5^* &=& (\alpha_4 r + \alpha_5 t)x^2 + 2(\alpha_6 r + \alpha_7 t)xq, \\ \alpha_6^* &=& \alpha_6 x^6, \\ \alpha_7^* &=& (\alpha_6 r + \alpha_7 t)x^3, \\ \alpha_8^* &=& \alpha_6 r^2 + 2\alpha_7 rt + \alpha_8 t^2. \end{array}$$

We interested in $(\alpha_2, \alpha_5, \alpha_7, \alpha_8) \neq (0,0,0,0)$ and $(\alpha_4, \alpha_6, \alpha_7) \neq (0,0,0)$. Let us consider the following cases:

1. $\alpha_6 = 0, \alpha_7 = 0$, then $\alpha_4 \neq 0$ and we have the following subcases:
   (a) $\alpha_8 = 0, \alpha_2\alpha_4 - \alpha_1\alpha_5 = 0$, then we have a split extension;
   (b) $\alpha_8 = 0, \alpha_2\alpha_4 - \alpha_1\alpha_5 \neq 0, \alpha_3 = 4\alpha_1$, then by choosing
   $$x = \sqrt[4]{\alpha_2\alpha_4 - \alpha_1\alpha_5}, t = \alpha_4^2, r = -\alpha_4\alpha_5, q = -\tfrac{\alpha_1 \sqrt[4]{\alpha_2\alpha_4 - \alpha_1\alpha_5}}{\alpha_4},$$
   we have the representative $\langle \nabla_2 + \nabla_4 \rangle$;
   (c) $\alpha_8 = 0, \alpha_2\alpha_4 - \alpha_1\alpha_5 \neq 0, \alpha_3 \neq 4\alpha_1$, then by choosing
   $$x = \tfrac{\alpha_3 - 4\alpha_1}{\alpha_4}, t = \tfrac{(\alpha_3 - 4\alpha_1)^4}{\alpha_4^2(\alpha_2\alpha_4 - \alpha_1\alpha_5)}, r = \tfrac{\alpha_5(\alpha_3 - 4\alpha_1)^4}{\alpha_4^3(\alpha_1\alpha_5 - \alpha_2\alpha_4)}, q = \tfrac{4\alpha_1^2 - \alpha_1\alpha_3}{\alpha_4^2},$$
   we have the representative $\langle \nabla_2 + \nabla_3 + \nabla_4 \rangle$;
   (d) $\alpha_8 \neq 0, \alpha_3 = 4\alpha_1$, then by choosing
   $$x = \alpha_4\alpha_8, t = \alpha_4^3\alpha_8^2, q = -\alpha_1\alpha_8, r = -\alpha_4^2\alpha_5\alpha_8^2, e = \alpha_1\alpha_5 - \alpha_2\alpha_4,$$
   we have the representative $\langle \nabla_4 + \nabla_8 \rangle$;
   (e) $\alpha_8 \neq 0, \alpha_3 \neq 4\alpha_1$, then by choosing
   $$x = \tfrac{\alpha_3 - 4\alpha_1}{\alpha_4},\ t = \tfrac{\sqrt{(\alpha_3 - 4\alpha_1)^5}}{\alpha_4^2\sqrt{\alpha_8}},\ q = \tfrac{4\alpha_1^2 - \alpha_1\alpha_3}{\alpha_4^2},\ r = -\tfrac{\alpha_5\sqrt{(\alpha_3 - 4\alpha_1)^5}}{\alpha_4^3\sqrt{\alpha_8}},$$
   $$e = \tfrac{(4\alpha_1 - \alpha_3)(\alpha_2\alpha_4 - \alpha_1\alpha_5)}{\alpha_4^2\alpha_8},$$
   we have the representative $\langle \nabla_3 + \nabla_4 + \nabla_8 \rangle$.

2. $\alpha_6 = 0, \alpha_7 \neq 0$, then we have the following subcases:
   (a) $\alpha_4 = 0, \alpha_3 = 0$, then by choosing
   $$x = 2\alpha_7^2,\ q = -\alpha_5\alpha_7,\ e = -2\alpha_1\alpha_7,\ w = \alpha_5^2 + 2\alpha_1\alpha_8 - 2\alpha_2\alpha_7,\ t = -2\alpha_7,\ r = \alpha_8,$$
   we have the representative $\langle \nabla_7 \rangle$;
   (b) $\alpha_4 = 0, \alpha_3 \neq 0$, then by choosing
   $$x = 1,\ q = -\frac{\alpha_5}{2\alpha_7},\ e = -\frac{\alpha_1}{\alpha_7},\ w = \frac{\alpha_5^2+2\alpha_1\alpha_8-2\alpha_2\alpha_7}{2\alpha_7^2},\ t = \frac{\alpha_3}{\alpha_7},\ r = -\frac{\alpha_3\alpha_8}{2\alpha_7^2},$$
   we have the representative $\langle \nabla_3 + \nabla_7 \rangle$;
   (c) $\alpha_4 \neq 0, \alpha_3\alpha_7^2 - 2\alpha_4\alpha_5\alpha_7 + \alpha_4^2\alpha_8 = 0$, then by choosing
   $$x = \sqrt{\alpha_7}, t = \alpha_4,\ e = \frac{\alpha_3-4\alpha_1}{4\sqrt{\alpha_7}},\ r = -\frac{\alpha_4\alpha_8}{2\alpha_7},\ q = -\frac{\alpha_3\sqrt{\alpha_7}}{4\alpha_4},$$
   $$w = \frac{4\alpha_1\alpha_4\alpha_8-4\alpha_2\alpha_4\alpha_7+\alpha_3(\alpha_5\alpha_7-\alpha_4\alpha_8)}{4\alpha_4\sqrt{\alpha_7^3}},$$
   we have the representative $\langle \nabla_4 + \nabla_7 \rangle$;
   (d) $\alpha_4 \neq 0, \alpha_3\alpha_7^2 - 2\alpha_4\alpha_5\alpha_7 + \alpha_4^2\alpha_8 \neq 0$, then by choosing
   $$x = -\frac{\alpha_3}{2\alpha_4} + \frac{\alpha_5}{\alpha_7} - \frac{\alpha_4\alpha_8}{2\alpha_7^2},\ q = \frac{\alpha_3(\alpha_3\alpha_7^2-2\alpha_4\alpha_5\alpha_7+\alpha_4^2\alpha_8)}{8\alpha_4^2\alpha_7^2},$$
   $$w = \frac{(\alpha_3\alpha_7^2-2\alpha_4\alpha_5\alpha_7+\alpha_4^2\alpha_8)(4\alpha_2\alpha_4\alpha_7-4\alpha_1\alpha_4\alpha_8+\alpha_3(-\alpha_5\alpha_7+\alpha_4\alpha_8))}{8\alpha_4^2\alpha_7^4},$$
   $$e = \frac{(4\alpha_1-\alpha_3)(\alpha_3\alpha_7^2-2\alpha_4\alpha_5\alpha_7+\alpha_4^2\alpha_8)}{8\alpha_4\alpha_7^3},\ t = \frac{(\alpha_3\alpha_7^2-2\alpha_4\alpha_5\alpha_7+\alpha_4^2\alpha_8)^2}{4\alpha_4\alpha_7^5},$$
   $$r = -\frac{\alpha_8(\alpha_3\alpha_7^2-2\alpha_4\alpha_5\alpha_7+\alpha_4^2\alpha_8)^2}{8\alpha_4\alpha_7^6},$$
   we have the representative $\langle \nabla_4 + \nabla_5 + \nabla_7 \rangle$.
3. $\alpha_6 \neq 0$, then we have the following subcases:
   (a) $\alpha_6\alpha_8 - \alpha_7^2 = 0, \alpha_5\alpha_6 - \alpha_4\alpha_7 = 0, \alpha_2\alpha_6 - \alpha_1\alpha_7 = 0$, then we have a split extension;
   (b) $\alpha_6\alpha_8 - \alpha_7^2 = 0, \alpha_5\alpha_6 - \alpha_4\alpha_7 = 0, \alpha_2\alpha_6 - \alpha_1\alpha_7 \neq 0, \alpha_3\alpha_6 - \alpha_4^2 = 0$, then by choosing
   $$x = 1, t = \frac{\alpha_6^2}{\alpha_2\alpha_6-\alpha_1\alpha_7}, q = -\frac{\alpha_4}{2\alpha_6}, r = \frac{\alpha_6\alpha_7}{\alpha_1\alpha_7-\alpha_2\alpha_6}, e = 0, w = \frac{\alpha_4^2-2\alpha_1\alpha_6}{\alpha_6},$$
   we have the representative $\langle \nabla_2 + \nabla_6 \rangle$;
   (c) $\alpha_6\alpha_8 - \alpha_7^2 = 0, \alpha_5\alpha_6 - \alpha_4\alpha_7 = 0, \alpha_2\alpha_6 - \alpha_1\alpha_7 \neq 0, \alpha_3\alpha_6 - \alpha_4^2 \neq 0$, then by choosing
   $$x = \sqrt{\frac{\alpha_3\alpha_6-\alpha_4^2}{\alpha_6^2}},\ t = \frac{\sqrt{(\alpha_3\alpha_6-\alpha_4^2)^5}}{\alpha_6^3(\alpha_2\alpha_6-\alpha_1\alpha_7)},\ q = -\frac{\alpha_4\sqrt{\alpha_3\alpha_6-\alpha_4^2}}{2\alpha_6^2},$$
   $$r = -\frac{\sqrt{(\alpha_3\alpha_6-\alpha_4^2)^5}\alpha_7}{\alpha_6^4(\alpha_2\alpha_6-\alpha_1\alpha_7)}, e = 0,$$
   and $w = \frac{(\alpha_4^2-2\alpha_1\alpha_6)\sqrt{\alpha_3\alpha_6-\alpha_4^2}}{2\alpha_6^3}$, we have the representative $\langle \nabla_2 + \nabla_3 + \nabla_6 \rangle$;
   (d) $\alpha_6\alpha_8 - \alpha_7^2 = 0, \alpha_5\alpha_6 - \alpha_4\alpha_7 \neq 0, 2\alpha_6(\alpha_2\alpha_6 - \alpha_1\alpha_7) = \alpha_4(\alpha_5\alpha_6 - \alpha_4\alpha_7)$, then by choosing
   $$t = \frac{\alpha_6^2}{\alpha_5\alpha_6-\alpha_4\alpha_7}x^4,\ q = -\frac{\alpha_4}{2\alpha_6}x,\ r = \frac{\alpha_6\alpha_7}{\alpha_4\alpha_7-\alpha_5\alpha_6}x^4,\ e = 0,\ w = \frac{\alpha_4^2-2\alpha_1\alpha_6}{\alpha_6}x,$$
   we have the representatives $\langle \nabla_5 + \nabla_6 \rangle$ and $\langle \nabla_3 + \nabla_5 + \nabla_6 \rangle$ depending on whether $\alpha_3\alpha_6 = \alpha_4^2$ or not;
   (e) $\alpha_6\alpha_8 - \alpha_7^2 = 0, \alpha_5\alpha_6 - \alpha_4\alpha_7 \neq 0, 2\alpha_6(\alpha_2\alpha_6 - \alpha_1\alpha_7) \neq \alpha_4(\alpha_5\alpha_6 - \alpha_4\alpha_7)$, then by choosing $x = \frac{2\alpha_6(\alpha_2\alpha_6-\alpha_1\alpha_7)-\alpha_4(\alpha_5\alpha_6-\alpha_4\alpha_7)}{2\alpha_6^2(\alpha_5\alpha_6-\alpha_4\alpha_7)}$, $t = \frac{\alpha_6^2}{\alpha_5\alpha_6-\alpha_4\alpha_7}x^4$, $q = -\frac{\alpha_4}{2\alpha_6}x$, $r = \frac{\alpha_6\alpha_7}{\alpha_4\alpha_7-\alpha_5\alpha_6}x^4$, $e = 0$, $w = \frac{\alpha_4^2-2\alpha_1\alpha_6}{\alpha_6}x$, we have the representative $\langle \nabla_2 + \alpha\nabla_3 + \nabla_5 + \nabla_6 \rangle$;
   (f) $\alpha_6\alpha_8 - \alpha_7^2 \neq 0, \alpha_5\alpha_6 - \alpha_4\alpha_7 = 0$, then by choosing

$$t=\frac{\alpha_6x^3}{\sqrt{\alpha_6\alpha_8-\alpha_7^2}},\ q=-\frac{\alpha_4x}{2\alpha_6},\ r=-\frac{\alpha_7x^3}{\sqrt{\alpha_6\alpha_8-\alpha_7^2}},\ e=\frac{(\alpha_1\alpha_7-\alpha_2\alpha_6)x}{\alpha_6\alpha_8-\alpha_7^2},$$
$$w=\Big(\frac{\alpha_4^2}{2\alpha_6^2}+\frac{\alpha_1\alpha_8-\alpha_2\alpha_7}{\alpha_7^2-\alpha_6\alpha_8}\Big)x,$$

we have the representatives $\langle\nabla_6+\nabla_8\rangle$ and $\langle\nabla_3+\nabla_6+\nabla_8\rangle$ depending on whether $\alpha_3\alpha_6=\alpha_4^2$ or not.

(g) $\alpha_6\alpha_8-\alpha_7^2\neq 0, \alpha_5\alpha_6-\alpha_4\alpha_7\neq 0$, then by choosing

$$x=\frac{\alpha_5\alpha_6-\alpha_4\alpha_7}{\sqrt{\alpha_6^2(\alpha_6\alpha_8-\alpha_7^2)}},\ t=\frac{(\alpha_5\alpha_6-\alpha_4\alpha_7)^3}{\alpha_6^2(\alpha_7^2-\alpha_6\alpha_8)^2},\ q=\frac{\alpha_4(\alpha_4\alpha_7-\alpha_5\alpha_6)}{2\alpha_6\sqrt{\alpha_6^2(\alpha_6\alpha_8-\alpha_7^2)}},$$
$$r=\frac{\alpha_7(\alpha_4\alpha_7-\alpha_5\alpha_6)^3}{\alpha_6^3(\alpha_7^2-\alpha_6\alpha_8)^2},\ e=\frac{\alpha_6(\alpha_5\alpha_6-\alpha_4\alpha_7)(\alpha_4\alpha_5\alpha_6-\alpha_4^2\alpha_7+2\alpha_6(-\alpha_2\alpha_6+\alpha_1\alpha_7))}{2\alpha_6^3\sqrt{(\alpha_6\alpha_8-\alpha_7^2)^3}},$$
$$w=\frac{\alpha_6(\alpha_5\alpha_6-\alpha_4\alpha_7)(\alpha_4^2\alpha_8-\alpha_4\alpha_5\alpha_7+2\alpha_6(\alpha_2\alpha_7-\alpha_1\alpha_8))}{2\alpha_6^3\sqrt{(\alpha_6\alpha_8-\alpha_7^2)^3}},$$

we have the representative $\langle\alpha\nabla_3+\nabla_5+\nabla_6+\nabla_8\rangle$.

Summarizing, we have the following distinct orbits

$$\langle\nabla_2+\nabla_3+\nabla_4\rangle,\ \langle\nabla_2+\alpha\nabla_3+\nabla_5+\nabla_6\rangle,\ \langle\nabla_2+\nabla_3+\nabla_6\rangle,\ \langle\nabla_2+\nabla_4\rangle,$$
$$\langle\nabla_2+\nabla_6\rangle,\ \langle\nabla_3+\nabla_4+\nabla_8\rangle,\ \langle\nabla_3+\nabla_5+\nabla_6\rangle,\ \langle\alpha\nabla_3+\nabla_5+\nabla_6+\nabla_8\rangle,$$
$$\langle\nabla_3+\nabla_6+\nabla_8\rangle,\ \langle\nabla_3+\nabla_7\rangle,\ \langle\nabla_4+\nabla_5+\nabla_7\rangle,\ \langle\nabla_4+\nabla_7\rangle,\ \langle\nabla_4+\nabla_8\rangle,$$
$$\langle\nabla_5+\nabla_6\rangle,\ \langle\nabla_6+\nabla_8\rangle,\ \langle\nabla_7\rangle,$$

which gives the following new algebras:

$\mathbf{N}_{31}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=e_5$	$e_2e_3=e_5$	
$\mathbf{N}_{32}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{33}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{34}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_3=e_5$		
$\mathbf{N}_{35}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{36}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{37}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_4=e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{38}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{39}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{40}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{41}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{42}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{43}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{44}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{45}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_3e_3=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{46}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_3e_4=e_5$			

3.2. **1-dimensional central extensions of $\mathbf{N}_{04}^{4*}$.** Here we will collect all information about $\mathbf{N}_{04}^{4*}$:

$\mathbf{N}_{04}^{4*}$	$e_1e_1=e_2$ $e_2e_2=e_3$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{04}^{4*}) = \langle[\Delta_{12}],[\Delta_{14}],[\Delta_{24}],[\Delta_{44}]\rangle$, $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{04}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{04}^{4*})\oplus \langle[\Delta_{13}],[\Delta_{23}],[\Delta_{33}],[\Delta_{34}]\rangle$	$\phi=\begin{pmatrix}x&0&0&0\\0&x^2&0&0\\y&0&x^4&r\\z&0&0&t\end{pmatrix}$

Let us use the following notations:

$$\nabla_1=[\Delta_{12}],\quad \nabla_2=[\Delta_{13}],\quad \nabla_3=[\Delta_{14}],\quad \nabla_4=[\Delta_{23}],$$
$$\nabla_5=[\Delta_{24}],\quad \nabla_6=[\Delta_{33}],\quad \nabla_7=[\Delta_{34}],\quad \nabla_8=[\Delta_{44}].$$

Take $\theta = \sum\limits_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^{4*}_{04})$. Since

$$\phi^T \begin{pmatrix} 0 & \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_1 & 0 & \alpha_4 & \alpha_5 \\ \alpha_2 & \alpha_4 & \alpha_6 & \alpha_7 \\ \alpha_3 & \alpha_5 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha_1^* & \alpha_2^* & \alpha_3^* \\ \alpha_1^* & \alpha^{**} & \alpha_4^* & \alpha_5^* \\ \alpha_2^* & \alpha_4^* & \alpha_6^* & \alpha_7^* \\ \alpha_3^* & \alpha_5^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

we have

$$\begin{array}{ll} \alpha_1^* = (\alpha_1 x + \alpha_4 y + \alpha_5 z)x^2, & \alpha_2^* = (\alpha_2 x + \alpha_6 y + \alpha_7 z)x^4, \\ \alpha_3^* = (\alpha_2 x + \alpha_6 y + \alpha_7 z)r + (\alpha_3 x + \alpha_7 y + \alpha_8 z)t, & \alpha_4^* = \alpha_4 x^6, \\ \alpha_5^* = (\alpha_4 r + \alpha_5 t)x^2, & \alpha_6^* = \alpha_6 x^8, \\ \alpha_7^* = (\alpha_6 r + \alpha_7 t)x^4, & \alpha_8^* = \alpha_6 r^2 + 2\alpha_7 rt + \alpha_8 t^2. \end{array}$$

We interested in $(\alpha_3, \alpha_5, \alpha_7, \alpha_8) \neq (0,0,0,0)$ and $(\alpha_2, \alpha_4, \alpha_6, \alpha_7) \neq (0,0,0,0)$. Let us consider the following cases:

1. $\alpha_6 = 0, \alpha_7 = 0, \alpha_4 = 0$, then $\alpha_2 \neq 0$ and we have the following cases:
   (a) if $\alpha_8 = 0, \alpha_5 = 0$, then by choosing $t = 1$ and $r = -\frac{\alpha_3}{\alpha_2}$, we have a split extension;
   (b) if $\alpha_8 = 0, \alpha_5 \neq 0$, then by choosing
   $$x = \alpha_2\alpha_5,\ t = \alpha_2^4\alpha_5^2,\ z = -\alpha_1\alpha_2,\ r = -\alpha_2^3\alpha_3\alpha_5^2,\ y = 0,$$
   we have the representative $\langle \nabla_2 + \nabla_5 \rangle$;
   (c) if $\alpha_8 \neq 0, \alpha_5 = 0, \alpha_1 = 0$, then by choosing
   $$x = \alpha_3,\ t = \sqrt{\alpha_2}\alpha_8^2,\ z = -\alpha_3,\ r = 0,\ y = 0,$$
   we have the representative $\langle \nabla_2 + \nabla_8 \rangle$;
   (d) if $\alpha_8 \neq 0, \alpha_5 = 0, \alpha_1 \neq 0$, then by choosing
   $$x = \sqrt{\alpha_1\alpha_2^{-1}},\ t = \sqrt[4]{\alpha_1^5\alpha_2^{-3}}\sqrt{\alpha_8^{-1}},\ z = -\sqrt{\alpha_1\alpha_2^{-1}}\alpha_3\alpha_8^{-1},\ r = 0,\ y = 0,$$
   we have the representative $\langle \nabla_1 + \nabla_2 + \nabla_8 \rangle$;
   (e) if $\alpha_8 \neq 0, \alpha_5 \neq 0$, then by choosing
   $$x = \frac{\alpha_5^2}{\alpha_2\alpha_8}, t = \frac{\alpha_5^5}{\alpha_2^2\alpha_8^3}, z = -\frac{\alpha_1\alpha_5}{\alpha_2\alpha_8}, r = \frac{\alpha_5^4(\alpha_1\alpha_8 - \alpha_3\alpha_5)}{\alpha_2^3\alpha_8^3}, y = 0,$$
   we have the representative $\langle \nabla_2 + \nabla_5 + \nabla_8 \rangle$.
2. $\alpha_6 = 0, \alpha_7 = 0, \alpha_4 \neq 0$, then by choosing $r = -\frac{\alpha_5}{\alpha_4}t, y = -\frac{\alpha_1 x + \alpha_5 z}{\alpha_4}$, we have $\alpha_1^* = \alpha_5^* = 0$. Now we can suppose that $\alpha_1 = 0, \alpha_5 = 0$, and we have the following subcases:
   (a) if $\alpha_8 = 0, \alpha_3 = 0$, then we have a split extension;
   (b) if $\alpha_8 = 0, \alpha_3 \neq 0, \alpha_2 = 0$, then by choosing $x = \alpha_3,\ y = 0,\ z = 0,\ r = 0,$ $t = \alpha_3^4$, we have the representative $\langle \nabla_3 + \nabla_4 \rangle$;
   (c) if $\alpha_8 = 0, \alpha_3 \neq 0, \alpha_2 \neq 0$, then by choosing $x = \frac{\alpha_2}{\alpha_4},\ y = 0,\ z = 0,$ $r = 0, t = \frac{\alpha_2^5}{\alpha_3\alpha_4^4}$, we have the representative $\langle \nabla_2 + \nabla_3 + \nabla_4 \rangle$;
   (d) if $\alpha_8 \neq 0, \alpha_2 = 0$, then by choosing $x = 1, y = 0, z = -\frac{\alpha_3}{\alpha_8}, r = 0, t = \sqrt{\frac{\alpha_4}{\alpha_8}}$, we have the representative $\langle \nabla_4 + \nabla_8 \rangle$;
   (e) if $\alpha_8 \neq 0, \alpha_2 \neq 0$, then by choosing $x = \frac{\alpha_3}{\alpha_4}, y = 0, z = -\frac{\alpha_2\alpha_3}{\alpha_4\alpha_8}, r = 0, t = \frac{\alpha_2^3}{\sqrt{\alpha_4^5\alpha_8}}$, we have the representative $\langle \nabla_2 + \nabla_4 + \nabla_8 \rangle$.
3. $\alpha_6 = 0, \alpha_7 \neq 0$, then by choosing $r = -\frac{\alpha_8 t}{2\alpha_7}, y = -\frac{(\alpha_2\alpha_8 - \alpha_3\alpha_7)x}{\alpha_7^2}, z = -\frac{\alpha_2 x}{\alpha_7}$, we have $\alpha_2^* = \alpha_3^* = \alpha_8^* = 0$. Now we can suppose that $\alpha_2 = 0,\ \alpha_3 = 0,\ \alpha_8 = 0,$ and consider the following cases:

(a) if $\alpha_4 = 0, \alpha_5 = 0, \alpha_1 = 0$, then we have the representative $\langle \nabla_7 \rangle$;
(b) if $\alpha_4 = 0, \alpha_5 = 0, \alpha_1 \neq 0$, then by choosing $x = \frac{1}{\alpha_7}, t = \alpha_1, y = 0, z = 0, r = 0$, we have the representative $\langle \nabla_1 + \nabla_7 \rangle$;
(c) if $\alpha_4 = 0, \alpha_5 \neq 0, \alpha_1 = 0$, then by choosing $x = \sqrt{\frac{\alpha_5}{\alpha_7}}, t = 1, y = 0, z = 0, r = 0$, we have the representative $\langle \nabla_5 + \nabla_7 \rangle$;
(d) if $\alpha_4 = 0, \alpha_5 \neq 0, \alpha_1 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_5}{\alpha_7}}, t = \sqrt{\frac{\alpha_1^2}{\alpha_5\alpha_7}}, y = 0, z = 0, r = 0$, we have the representative $\langle \nabla_1 + \nabla_5 + \nabla_7 \rangle$;
(e) if $\alpha_4 \neq 0, \alpha_5 = 0, \alpha_1 = 0$, then by choosing $x = \sqrt{\alpha_7}, t = \alpha_4, y = 0, z = 0, r = 0$, we have the representative $\langle \nabla_4 + \nabla_7 \rangle$;
(f) if $\alpha_4 \neq 0, \alpha_5 = 0, \alpha_1 \neq 0$, then by choosing $x = \sqrt[3]{\frac{\alpha_1}{\alpha_4}}, t = \sqrt[3]{\frac{\alpha_1\alpha_4^2}{\alpha_7^3}}, y = 0, z = 0, r = 0$, we have the representative $\langle \nabla_1 + \nabla_4 + \nabla_7 \rangle$;
(g) if $\alpha_4 \neq 0, \alpha_5 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_5}{\alpha_7}}, t = \sqrt{\frac{\alpha_4\alpha_5}{\alpha_7^2}}, y = 0, z = 0, r = 0$, we have the representative $\langle \alpha\nabla_1 + \nabla_4 + \nabla_5 + \nabla_7 \rangle$.

4. $\alpha_6 \neq 0$, then by choosing $r = -\frac{\alpha_7 t}{\alpha_6}, y = -\frac{\alpha_2 x + \alpha_7 z}{\alpha_6}$, we have $\alpha_2^* = \alpha_7^* = 0$. Now we can suppose that $\alpha_2 = 0, \alpha_7 = 0$, and we have:
(a) if $\alpha_8 = 0, \alpha_5 = 0$, then $\alpha_3 \neq 0$ and we have the following subcases:
  (i) $\alpha_4 = 0, \alpha_1 = 0$, then by choosing $x = \alpha_3, t = \alpha_3^6\alpha_6, y = 0, z = 0, r = 0$, we have the representative $\langle \nabla_3 + \nabla_6 \rangle$;
  (ii) $\alpha_4 = 0, \alpha_1 \neq 0$, then by choosing $x = \sqrt[5]{\frac{\alpha_1}{\alpha_6}}, t = \sqrt[5]{\frac{\alpha_1^7}{\alpha_3^5\alpha_6^2}}, y = 0, z = 0, r = 0$, we have the representative $\langle \nabla_1 + \nabla_3 + \nabla_6 \rangle$;
  (iii) $\alpha_4 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_4}{\alpha_6}}, t = \sqrt[5]{\frac{\alpha_4^7}{\alpha_3^2\alpha_6^5}}, y = 0, z = 0, r = 0$, we have the representative $\langle \alpha\nabla_1 + \nabla_3 + \nabla_4 + \nabla_6 \rangle$.
(b) $\alpha_8 = 0, \alpha_5 \neq 0$, then we have the following subcases:
  (i) $\alpha_4 = 0, \alpha_3 = 0$, then by choosing $x = \sqrt[6]{\frac{\alpha_5}{\alpha_6}}, t = 1, z = -\frac{\alpha_1}{\sqrt[6]{\alpha_5^5\alpha_6}}, y = 0, r = 0$, we have the representative $\langle \nabla_5 + \nabla_6 \rangle$;
  (ii) $\alpha_4 = 0, \alpha_3 \neq 0$, then by choosing $x = \frac{\alpha_3}{\alpha_5}, t = \alpha_3^6\alpha_5^{-7}\alpha_6, z = -\frac{\alpha_1\alpha_3}{\alpha_5^2}, y = 0, r = 0$, we have the representative $\langle \nabla_3 + \nabla_5 + \nabla_6 \rangle$;
  (iii) $\alpha_4 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_4}{\alpha_6}}, t = \frac{\alpha_4^3}{\alpha_5\alpha_6^2}, z = -\alpha_1\alpha_5^{-1}\sqrt{\alpha_4\alpha_6^{-1}}, y = 0, r = 0$, we have the representative $\langle \alpha\nabla_3 + \nabla_4 + \nabla_5 + \nabla_6 \rangle$.
(c) $\alpha_8 \neq 0$, then we have the following subcases:
  (i) $\alpha_5 = 0, \alpha_4 = 0, \alpha_1 = 0$, then by choosing $x = 1, t = \sqrt{\frac{\alpha_6}{\alpha_8}}, z = -\frac{\alpha_3}{\alpha_8}, y = 0, r = 0$, we have the representative $\langle \nabla_6 + \nabla_8 \rangle$;
  (ii) $\alpha_5 = 0, \alpha_4 = 0, \alpha_1 \neq 0$, then by choosing
$$x = \sqrt[5]{\alpha_1\alpha_6^{-1}},\ t = \sqrt[10]{\alpha_1^8\alpha_6^{-3}\alpha_8^{-5}},\ z = -\alpha_3\alpha_8^{-1}\sqrt[5]{\alpha_1\alpha_6^{-1}},\ y = 0,\ r = 0,$$
  we have the representative $\langle \nabla_1 + \nabla_6 + \nabla_8 \rangle$;
  (iii) $\alpha_5 = 0, \alpha_4 \neq 0$, then by choosing
$$x = \sqrt{\alpha_4\alpha_6^{-1}},\ t = \alpha_4^2\sqrt{\alpha_6^{-3}\alpha_8^{-1}},\ z = -\alpha_3\alpha_8^{-1}\sqrt{\alpha_4\alpha_6^{-1}},\ y = 0,\ r = 0,$$
  we have the representative $\langle \alpha\nabla_1 + \nabla_4 + \nabla_6 + \nabla_8 \rangle$;
  (iv) $\alpha_5 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_5^2}{\alpha_6\alpha_8}},\ t = \frac{\alpha_5^2}{\sqrt{\alpha_6\alpha_8^3}},\ z = -\frac{\alpha_3\sqrt{\alpha_5}}{\sqrt[4]{\alpha_6\alpha_8^5}}$, $y = 0,\ r = 0$, we have the representative $\langle \alpha\nabla_1 + \beta\nabla_4 + \nabla_5 + \nabla_6 + \nabla_8 \rangle$.

Summarizing, we have the following distinct orbits:

$$\langle\nabla_1+\nabla_2+\nabla_8\rangle, \langle\nabla_1+\nabla_3+\nabla_6\rangle, \langle\alpha\nabla_1+\nabla_3+\nabla_4+\nabla_6\rangle^{O(\alpha)=O(-\alpha)},$$
$$\langle\alpha\nabla_1+\beta\nabla_4+\nabla_5+\nabla_6+\nabla_8\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)=O(\pm i\alpha,-\beta)},$$
$$\langle\alpha\nabla_1+\nabla_4+\nabla_6+\nabla_8\rangle^{O(\alpha)=O(-\alpha)}, \langle\nabla_1+\nabla_4+\nabla_7\rangle,$$
$$\langle\alpha\nabla_1+\nabla_4+\nabla_5+\nabla_7\rangle^{O(\alpha)=O(-\alpha)}, \langle\nabla_1+\nabla_5+\nabla_7\rangle, \langle\nabla_1+\nabla_6+\nabla_8\rangle,$$
$$\langle\nabla_1+\nabla_7\rangle, \langle\nabla_2+\nabla_3+\nabla_4\rangle, \langle\nabla_2+\nabla_4+\nabla_8\rangle, \langle\nabla_2+\nabla_5\rangle, \langle\nabla_2+\nabla_5+\nabla_8\rangle,$$
$$\langle\nabla_2+\nabla_8\rangle, \langle\nabla_3+\nabla_4\rangle, \langle\alpha\nabla_3+\nabla_4+\nabla_5+\nabla_6\rangle^{O(\alpha)=O(-\alpha)}, \langle\nabla_3+\nabla_5+\nabla_6\rangle,$$
$$\langle\nabla_3+\nabla_6\rangle, \langle\nabla_4+\nabla_7\rangle, \langle\nabla_4+\nabla_8\rangle, \langle\nabla_5+\nabla_6\rangle, \langle\nabla_5+\nabla_7\rangle, \langle\nabla_6+\nabla_8\rangle, \langle\nabla_7\rangle.$$

Hence, we have the following new algebras:

$\mathbf{N}_{47}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_4e_4=e_5$	
$\mathbf{N}_{48}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_3e_3=e_5$	
$\mathbf{N}_{49}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=\alpha e_5$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{50}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=\alpha e_5$	$e_2e_2=e_3$	$e_2e_3=\beta e_5$		
		$e_2e_4=e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$			
$\mathbf{N}_{51}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=\alpha e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{52}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{53}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=\alpha e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_4=e_5$
$\mathbf{N}_{54}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{55}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_2e_2=e_3$	$e_3e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{56}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_2e_2=e_3$	$e_3e_4=e_5$		
$\mathbf{N}_{57}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$	
$\mathbf{N}_{58}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{59}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_2e_4=e_5$		
$\mathbf{N}_{60}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{61}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_4e_4=e_5$		
$\mathbf{N}_{62}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$		
$\mathbf{N}_{63}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{64}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{65}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_3e_3=e_5$		
$\mathbf{N}_{66}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{67}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{68}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{69}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{70}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_3e_3=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{71}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_3e_4=e_5$			

3.3. **1-dimensional central extensions of $\mathbf{N}_{08}^{4*}$.** Here we will collect all information about $\mathbf{N}_{08}^{4*}$:

$\mathbf{N}_{08}^{4*}$	$e_1e_1=e_2$ $e_1e_2=e_3$ $e_2e_2=e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{08}^{4*}) = \langle[\Delta_{13}],[\Delta_{14}]+3[\Delta_{23}]\rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{08}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{08}^{4*})\oplus \langle[\Delta_{14}],[\Delta_{24}],[\Delta_{33}],[\Delta_{34}],[\Delta_{44}]\rangle$	$\phi=\begin{pmatrix} x & 0 & 0 & 0\\ y & x^2 & 0 & 0\\ z & 2xy & x^3 & 0\\ t & y^2 & x^2y & x^4\end{pmatrix}$

Let us use the following notations:

$$\nabla_1=[\Delta_{13}],\quad \nabla_2=[\Delta_{14}]+3[\Delta_{23}],\quad \nabla_3=[\Delta_{14}],\quad \nabla_4=[\Delta_{24}],$$
$$\nabla_5=[\Delta_{33}],\quad \nabla_6=[\Delta_{34}],\quad \nabla_7=[\Delta_{44}].$$

Take $\theta = \sum\limits_{i=1}^{7}\alpha_i\nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^{4*}_{08})$. Since

$$\phi^T\begin{pmatrix} 0 & 0 & \alpha_1 & \alpha_2+\alpha_3\\ 0 & 0 & 3\alpha_2 & \alpha_4\\ \alpha_1 & 3\alpha_2 & \alpha_5 & \alpha_6\\ \alpha_2+\alpha_3 & \alpha_4 & \alpha_6 & \alpha_7\end{pmatrix}\phi = \begin{pmatrix} \alpha^* & \alpha^{**} & \alpha_1^* & \alpha_2^*+\alpha_3^*\\ \alpha^{**} & \alpha^{***} & 3\alpha_2^* & \alpha_4^*\\ \alpha_1^* & 3\alpha_2^* & \alpha_5^* & \alpha_6^*\\ \alpha_2^*+\alpha_3^* & \alpha_4^* & \alpha_6^* & \alpha_7^*\end{pmatrix},$$

we have

$$\begin{array}{rcl}
\alpha_1^* &=& (\alpha_1x+3\alpha_2y+\alpha_5z+\alpha_6t)x^3+((\alpha_2+\alpha_3)x+\alpha_4y+\alpha_6z+\alpha_7t)x^2y,\\
\alpha_2^* &=& \frac{1}{3}(3\alpha_2x^3+(\alpha_4+2\alpha_5)x^2y+3\alpha_6xy^2+\alpha_7y^3)x^2,\\
\alpha_3^* &=& ((\alpha_2+\alpha_3)x+\alpha_4y+\alpha_6z+\alpha_7t)x^4-\\
&& \frac{1}{3}(3\alpha_2x^3+(\alpha_4+2\alpha_5)x^2y+3\alpha_6xy^2+\alpha_7y^3)x^2,\\
\alpha_4^* &=& (\alpha_4x^2+2\alpha_6xy+\alpha_7y^2)x^4,\\
\alpha_5^* &=& (\alpha_5x^2+2\alpha_6xy+\alpha_7y^2)x^4,\\
\alpha_6^* &=& (\alpha_6x+\alpha_7y)x^6,\\
\alpha_7^* &=& \alpha_7x^8.
\end{array}$$

We are interested in $(\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7)\neq(0,0,0,0,0)$, $(\alpha_2+\alpha_3,\alpha_4,\alpha_6,\alpha_7)\neq(0,0,0,0)$ and $(\alpha_1,\alpha_2,\alpha_5,\alpha_6)\neq(0,0,0,0)$. Let us consider the following cases:

1. $\alpha_7=0,\alpha_6=0,\alpha_5=0,\alpha_4=0$, then $\alpha_3\neq0$, $\alpha_2+\alpha_3\neq0$ and $(\alpha_1,\alpha_2)\neq(0,0)$.
   (a) if $\alpha_2\neq-\frac{\alpha_3}{4}$, then by choosing $x=4\alpha_2+\alpha_3, y=-\alpha_1$, we have the representative $\langle\alpha\nabla_2+\nabla_3\rangle_{\alpha\neq0,-\frac{1}{4},-1}$;
   (b) if $\alpha_2=-\frac{\alpha_3}{4}$, then we have the representatives
   $$\left\langle-\tfrac{1}{4}\nabla_2+\nabla_3\right\rangle \text{ and } \left\langle\nabla_1-\tfrac{1}{4}\nabla_2+\nabla_3\right\rangle$$
   depending on $\alpha_1=0$ or not.
2. $\alpha_7=0,\alpha_6=0,\alpha_5=0,\alpha_4\neq0$, then by choosing $y=-\frac{3\alpha_2}{\alpha_4}x$, we have $\alpha_2^*=0$. This we can suppose $\alpha_2=0$, which implies $\alpha_1\neq0$ and choosing $x=\sqrt{\alpha_1\alpha_4^{-1}}$, we have the representative $\langle\nabla_1+\alpha\nabla_3+\nabla_4\rangle$.
3. $\alpha_7=0,\alpha_6=0,\alpha_5\neq0$.
   (a) if $\alpha_4=0$, then $\alpha_2\neq-\alpha_3$ and choosing
   $$x=\tfrac{\alpha_2+\alpha_3}{\alpha_5},\ y=\tfrac{3\alpha_2\alpha_3+3\alpha_3^2}{2\alpha_5^2},\ z=-\tfrac{(\alpha_2+\alpha_3)(2\alpha_1\alpha_5+12\alpha_2\alpha_3+3\alpha_3^2)}{4\alpha_5^3},$$
   we have the representative $\langle\nabla_2+\nabla_5\rangle$;
   (b) if $\alpha_4\neq0,\alpha_4\neq\alpha_5, 2(\alpha_2\alpha_5-\alpha_2\alpha_4+\alpha_3\alpha_5)+\alpha_3\alpha_4=0$, then by choosing
   $$x=2(\alpha_4-\alpha_5), y=3\alpha_3, z=0, t=0,$$
   we have the representative $\langle\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq0,1}$;
   (c) if $\alpha_4\neq0,\alpha_4\neq\alpha_5, 2(\alpha_2\alpha_5-\alpha_2\alpha_4+\alpha_3\alpha_5)+\alpha_3\alpha_4\neq0$, then by choosing
   $$x=\tfrac{2(\alpha_2\alpha_5-\alpha_2\alpha_4+\alpha_3\alpha_5)+\alpha_3\alpha_4}{2(\alpha_5^2-\alpha_4\alpha_5)},\ y=\tfrac{3\alpha_3(2(\alpha_2\alpha_5-\alpha_2\alpha_4+\alpha_3\alpha_5)+\alpha_3\alpha_4)}{2\alpha_5(\alpha_5-\alpha_4)^2},\ z=$$
   $$-\tfrac{(2\alpha_2(\alpha_4-\alpha_5)-\alpha_3(\alpha_4+2\alpha_5))(4\alpha_1(\alpha_4-\alpha_5)^2-24\alpha_2\alpha_3(\alpha_4-\alpha_5)+3\alpha_3^2(\alpha_4+2\alpha_5))}{8(\alpha_4-\alpha_5)^3\alpha_5^2}, t=0,$$
   we have the family of representatives $\langle\nabla_2+\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq0,1}$;
   (d) if $\alpha_4\neq0,\alpha_4=\alpha_5$, then by choosing $y=-\frac{\alpha_2x}{\alpha_5}$ and $z=\frac{(\alpha_3\alpha_5-\alpha_1\alpha_5+3\alpha_2^2)x}{\alpha_5^2}$, we have the representatives $\langle\nabla_4+\nabla_5\rangle$ and $\langle\nabla_3+\nabla_4+\nabla_5\rangle$ depending on whether $\alpha_3=0$ or not. Note that $\langle\nabla_4+\nabla_5\rangle=\langle\nabla_2+\nabla_4+\nabla_5\rangle$ and it will be jointed with the family from the case (3c).
4. if $\alpha_7=0,\alpha_6\neq0$, then by choosing $x=1$, $y=\frac{\sqrt{(\alpha_4+2\alpha_5)^2-36\alpha_2\alpha_6}-\alpha_4-2\alpha_5}{6\alpha_6}$,
   $$z=y^2-\tfrac{\alpha_3}{\alpha_6}+\tfrac{2y(\alpha_5-\alpha_4)}{3\alpha_6} \text{ and } t=-\tfrac{x^2\alpha_1+xy(4\alpha_2+\alpha_3)+xz\alpha_5+y(y\alpha_4+z\alpha_6)}{\alpha_6)},$$

we have $\alpha_1^*=\alpha_2^*=\alpha_3^*=0$. Now we can suppose that $\alpha_1=0,\alpha_2=0,\alpha_3=0$, and we have the following cases:

(a) if $\alpha_4=0,\alpha_5=0$, then by choosing $x=1,y=0,z=0,t=0$, we have the representative $\langle\nabla_6\rangle$;

(b) if $\alpha_4=0,\alpha_5\neq 0$, then by choosing $x=-\frac{4\alpha_5}{3\alpha_6},y=\frac{8\alpha_5^2}{9\alpha_6^2},z=0,t=0$, we have the representative $\left\langle\nabla_4+\frac{1}{4}\nabla_5+\nabla_6\right\rangle$;

(c) if $\alpha_4\neq 0$, then by choosing $x=\frac{\alpha_4}{\alpha_6},y=0,z=0,t=0$, we have the family of representatives $\langle\nabla_4+\alpha\nabla_5+\nabla_6\rangle$, which will be jointed with the representative from the case (4b).

5. if $\alpha_7\neq 0$, then by choosing $x=1$, $y=-\frac{\alpha_6}{\alpha_7}$, $t=\frac{2\alpha_6^3+2(\alpha_4-\alpha_5)\alpha_6\alpha_7-3\alpha_3\alpha_7^2}{3\alpha_7^3}$ and $z=0$, we have $\alpha_3^*=0,\alpha_6^*=0$. Now we can suppose that $\alpha_3=0,\alpha_6=0$, and we have the following cases:

(a) if $\alpha_5\neq 0$, then by choosing $x=\sqrt{\alpha_5\alpha_7^{-1}},y=0,z=\sqrt{\alpha_1^2\alpha_5^{-1}\alpha_7^{-1}},t=0$, we have the family of representatives $\langle\alpha\nabla_2+\beta\nabla_4+\nabla_5+\nabla_7\rangle$;

(b) if $\alpha_5=0,\alpha_2=0$, then $\alpha_1\neq 0$ and we have the family of representatives $\langle\nabla_1+\alpha\nabla_4+\nabla_7\rangle$;

(c) if $\alpha_5=0,\alpha_2\neq 0$, then by choosing $x=\sqrt[3]{\alpha_2\alpha_7^{-1}},y=0,z=0,t=0$, we have the family of representatives $\langle\alpha\nabla_1+\nabla_2+\beta\nabla_4+\nabla_7\rangle$.

Summarizing all cases we have the following distinct orbits

$$\left\langle\nabla_1-\tfrac{1}{4}\nabla_2+\nabla_3\right\rangle,\ \langle\alpha\nabla_1+\nabla_2+\beta\nabla_4+\nabla_7\rangle^{O(\alpha,\beta)=O(-\eta_3\alpha,\eta_3^2\beta)=O(\eta_3^2\alpha,-\eta_3\beta)},$$
$$\langle\nabla_1+\alpha\nabla_3+\nabla_4\rangle^{O(\alpha)=O(-\alpha)},\ \langle\nabla_1+\alpha\nabla_4+\nabla_7\rangle^{O(\alpha)=O(-\alpha)},\ \langle\alpha\nabla_2+\nabla_3\rangle_{\alpha\neq 0,-1},$$
$$\langle\nabla_2+\alpha\nabla_4+\nabla_5\rangle,\ \langle\alpha\nabla_2+\beta\nabla_4+\nabla_5+\nabla_7\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)},\ \langle\nabla_3+\nabla_4+\nabla_5\rangle,$$
$$\langle\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq 0,1},\ \langle\nabla_4+\alpha\nabla_5+\nabla_6\rangle,\ \langle\nabla_6\rangle,$$

which gives the following new algebras:

$\mathbf{N}_{72}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$		
		$e_1e_4=\frac{3}{4}e_5$	$e_2e_2=e_4$	$e_2e_3=-\frac{3}{4}e_5$		
$\mathbf{N}_{73}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$	
		$e_2e_2=e_4$	$e_2e_3=3e_5$	$e_2e_4=\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{74}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$		
		$e_1e_4=\alpha e_5$	$e_2e_2=e_4$	$e_2e_4=e_5$		
$\mathbf{N}_{75}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$		
		$e_2e_2=e_4$	$e_2e_4=\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{76}^{\alpha\neq 0,-1}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=(1+\alpha)e_5$		
		$e_2e_2=e_4$	$e_2e_3=3\alpha e_5$			
$\mathbf{N}_{77}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=e_4$	
		$e_2e_3=3e_5$	$e_2e_4=\alpha e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{78}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=e_4$	
		$e_2e_3=3\alpha e_5$	$e_2e_4=\beta e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{79}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$		
		$e_2e_2=e_4$	$e_2e_4=e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{80}^{\alpha\neq 0,1}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_4$	$e_2e_4=\alpha e_5$	$e_3e_3=e_5$
$\mathbf{N}_{81}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_4$		
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{82}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_4$	$e_3e_4=e_5$	

3.4. **1-dimensional central extensions of $\mathbf{N}_{09}^{4*}$.** Here we will collect all information about $\mathbf{N}_{09}^{4*}$:

$\mathbf{N}_{09}^{4*}$	$e_1e_1 = e_2$ $e_2e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{09}^{4*}) = \langle[\Delta_{12}],[\Delta_{13}],[\Delta_{22}],[\Delta_{33}]\rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{09}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{09}^{4*})\oplus \langle[\Delta_{14}],[\Delta_{24}],[\Delta_{34}],[\Delta_{44}]\rangle$	$\phi=\begin{pmatrix} x & 0 & 0 & 0\\ 0 & x^2 & 0 & 0\\ 0 & 0 & r & 0\\ t & 0 & s & x^2r\end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1=[\Delta_{12}], & \nabla_2=[\Delta_{13}], & \nabla_3=[\Delta_{14}], & \nabla_4=[\Delta_{22}],\\ \nabla_5=[\Delta_{24}], & \nabla_6=[\Delta_{33}], & \nabla_7=[\Delta_{34}], & \nabla_8=[\Delta_{44}]. \end{array}$$

Take $\theta=\sum\limits_{i=1}^{8}\alpha_i\nabla_i\in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{09}^{4*})$. Since

$$\phi^T\begin{pmatrix} 0 & \alpha_1 & \alpha_2 & \alpha_3\\ \alpha_1 & \alpha_4 & 0 & \alpha_5\\ \alpha_2 & 0 & \alpha_6 & \alpha_7\\ \alpha_3 & \alpha_5 & \alpha_7 & \alpha_8\end{pmatrix}\phi=\begin{pmatrix} \alpha^* & \alpha_1^* & \alpha_2^* & \alpha_3^*\\ \alpha_1^* & \alpha_4^* & \alpha^{**} & \alpha_5^*\\ \alpha_1^* & \alpha^{**} & \alpha_6^* & \alpha_7^*\\ \alpha_3^* & \alpha_5^* & \alpha_7^* & \alpha_8^*\end{pmatrix},$$

we have

$$\begin{array}{ll} \alpha_1^*=(\alpha_1x+\alpha_5t)x^2, & \alpha_2^*=(\alpha_2x+\alpha_7t)r+(\alpha_3x+\alpha_8t)s,\\ \alpha_3^*=(\alpha_3x+\alpha_8t)x^2r, & \alpha_4^*=\alpha_4x^4,\\ \alpha_5^*=\alpha_5x^4r, & \alpha_6^*=(\alpha_6r+\alpha_7s)r+(\alpha_7r+\alpha_8s)s,\\ \alpha_7^*=(\alpha_7r+\alpha_8s)x^2r, & \alpha_8^*=\alpha_8r^2x^4. \end{array}$$

We are interested in $(\alpha_3,\alpha_5,\alpha_7,\alpha_8)\neq(0,0,0,0)$. Let us consider the following cases:

1. $\alpha_8=0,\alpha_7=0,\alpha_5=0$, then $\alpha_3\neq0$ and we have
   (a) if $\alpha_1=0,\alpha_4=0,\alpha_6=0$, then by choosing $x=1,r=\alpha_3,s=-\alpha_2,t=0$, we have the representative $\langle\nabla_3\rangle$;
   (b) if $\alpha_1=0,\alpha_4=0,\alpha_6\neq0$, then by choosing $x=\alpha_6,r=\alpha_3\alpha_6^2,s=-\alpha_2\alpha_6^2,t=0$, we have the representative $\langle\nabla_3+\nabla_6\rangle$;
   (c) if $\alpha_1=0,\alpha_4\neq0,\alpha_6=0$, then by choosing $x=\alpha_3^2,r=\alpha_3\alpha_4,s=-\alpha_2\alpha_4,t=0$, we have the representative $\langle\nabla_3+\nabla_4\rangle$;
   (d) if $\alpha_1=0,\alpha_4\neq0,\alpha_6\neq0$, then by choosing $x={\alpha_3}^{-1}\sqrt{\alpha_4\alpha_6}$, $r={\alpha_3}^{-2}\sqrt{\alpha_4^3\alpha_6}$, $s=-\alpha_2{\alpha_3}^{-3}\sqrt{\alpha_4^3\alpha_6}$, $t=0$, we have the representative $\langle\nabla_3+\nabla_4+\nabla_6\rangle$;
   (e) if $\alpha_1\neq0,\alpha_4=0,\alpha_6=0$, then by choosing $x=1$, $r=\alpha_1{\alpha_3}^{-1}$, $s=-\alpha_1\alpha_2\alpha_3^{-2}$, $t=0$, we have the representative $\langle\nabla_1+\nabla_3\rangle$;
   (f) if $\alpha_1\neq0,\alpha_4=0,\alpha_6\neq0$, then by choosing $x=\sqrt[3]{\alpha_1\alpha_6\alpha_3^{-2}}$, $r=\alpha_1\alpha_3^{-1}$, $s=-\alpha_1\alpha_2\alpha_3^{-2}$, $t=0$, we have the representative $\langle\nabla_1+\nabla_3+\nabla_6\rangle$;
   (g) if $\alpha_1\neq0,\alpha_4\neq0$, then by choosing
   $$x=\alpha_1\alpha_4^{-1},\ r=\alpha_1\alpha_3^{-1},\ s=-\alpha_1\alpha_2\alpha_3^{-2},\ t=0,$$
   we have the family of representatives $\langle\nabla_1+\nabla_3+\nabla_4+\alpha\nabla_6\rangle$.
2. $\alpha_8=0,\alpha_7=0,\alpha_5\neq0$ and we have
   (a) if $\alpha_3=0,\alpha_2=0,\alpha_4=0,\alpha_6=0$, then by choosing $r=1,x=\alpha_5,t=-\alpha_1,s=0$, we have the representative $\langle\nabla_5\rangle$;
   (b) if $\alpha_3=0,\alpha_2=0,\alpha_4=0,\alpha_6\neq0$, then by choosing $x=\alpha_5\alpha_6$, $r=\alpha_5^5\alpha_6^3,s=0,t=-\alpha_1\alpha_6$, we have the representative $\langle\nabla_5+\nabla_6\rangle$;

(c) if $\alpha_3=0, \alpha_2=0, \alpha_4\neq 0, \alpha_6=0$, then by choosing $x=1, r=\alpha_4{\alpha_5}^{-1}, t=-\alpha_1{\alpha_5}^{-1}, s=0$, we have the representative $\langle \nabla_4+\nabla_5\rangle$;

(d) if $\alpha_3=0, \alpha_2=0, \alpha_4\neq 0, \alpha_6\neq 0$, then by choosing

$$x=\sqrt[4]{\alpha_4\alpha_6\alpha_5^{-2}},\ r=\alpha_4{\alpha_5}^{-1},\ t=-\alpha_1\sqrt[4]{\alpha_4\alpha_6\alpha_5^{-6}}, s=0,$$

we have the representative $\langle \nabla_4+\nabla_5+\nabla_6\rangle$;

(e) if $\alpha_3=0, \alpha_2\neq 0, \alpha_4=0, \alpha_6=0$, then by choosing

$$r=1, x=\sqrt[3]{\alpha_2{\alpha_5}^{-1}}, t=-\alpha_1\sqrt[3]{\alpha_2\alpha_5^{-4}}, s=0,$$

we have the representative $\langle \nabla_2+\nabla_5\rangle$;

(f) if $\alpha_3=0, \alpha_2\neq 0, \alpha_4=0, \alpha_6\neq 0$, then by choosing

$$x=\sqrt[3]{\alpha_2{\alpha_5}^{-1}},\ r=\alpha_6^{-1}\sqrt[3]{\alpha_2^4\alpha_5^{-1}},\ t=-\alpha_1\sqrt[3]{\alpha_2\alpha_5^{-4}}, s=0,$$

we have the representative $\langle \nabla_2+\nabla_5+\nabla_6\rangle$;

(g) if $\alpha_3=0, \alpha_2\neq 0, \alpha_4\neq 0$, then by choosing

$$x=\sqrt[3]{\alpha_2\alpha_5^{-1}}, r=\alpha_4{\alpha_5}^{-1}, t=-\alpha_1\sqrt[3]{\alpha_2\alpha_5^{-4}}, s=0,$$

we have the family of representatives $\langle \nabla_2+\nabla_4+\nabla_5+\alpha\nabla_6\rangle$;

(h) if $\alpha_3\neq 0, \alpha_4=0, \alpha_6=0$, then by choosing $x=\alpha_3{\alpha_5}^{-1}, r=\alpha_3, s=-\alpha_2, t=-\alpha_1\alpha_3\alpha_5^{-2}$, we have the representative $\langle \nabla_3+\nabla_5\rangle$;

(i) if $\alpha_3\neq 0, \alpha_4=0, \alpha_6\neq 0$, then by choosing

$$x=\alpha_3{\alpha_5}^{-1}, r=\alpha_3^4\alpha_5^{-3}\alpha_6^{-1}, s=-\alpha_2\alpha_3^3\alpha_5^{-3}\alpha_6^{-1}, t=-\alpha_1\alpha_3\alpha_5^{-2},$$

we have the representative $\langle \nabla_3+\nabla_5+\nabla_6\rangle$;

(j) if $\alpha_3\neq 0, \alpha_4\neq 0$, then by choosing $x=\alpha_3{\alpha_5}^{-1}, r=\alpha_4{\alpha_5}^{-1}, s=-\alpha_2\alpha_4\alpha_3^{-1}\alpha_5^{-1}, t=-\alpha_1\alpha_3\alpha_5^{-2}$, we have the family of representatives $\langle \nabla_3+\nabla_4+\nabla_5+\alpha\nabla_6\rangle$.

3. $\alpha_8=0, \alpha_7\neq 0$, then by choosing $x=2\alpha_7^2$, $t=\alpha_3\alpha_6-2\alpha_2\alpha_7$, $s=-\alpha_6$, $r=2\alpha_7$, we have $\alpha_2^*=\alpha_6^*=0$. Now we can suppose that $\alpha_2=0$ and $\alpha_6=0$, then for $s=0$ and $t=0$, we have:

(a) if $\alpha_1=0, \alpha_3=0, \alpha_4=0, \alpha_5=0$, then by choosing $r=1, x=1$, we have the representative $\langle \nabla_7\rangle$;

(b) if $\alpha_1=0, \alpha_3=0, \alpha_4=0, \alpha_5\neq 0$, then by choosing $x=\alpha_7, r=\alpha_5\alpha_7$, we have the representative $\langle \nabla_5+\nabla_7\rangle$;

(c) if $\alpha_1=0, \alpha_3=0, \alpha_4\neq 0, \alpha_5=0$, then by choosing $x=\sqrt{\alpha_7}, r=\sqrt{\alpha_4}$, we have the representative $\langle \nabla_4+\nabla_7\rangle$;

(d) if $\alpha_1=0, \alpha_3=0, \alpha_4\neq 0, \alpha_5\neq 0$, then by choosing $x=\alpha_4\sqrt{\alpha_7\alpha_5^{-3}}, r=\alpha_4{\alpha_5}^{-1}$, we have the representative $\langle \nabla_4+\nabla_5+\nabla_7\rangle$;

(e) if $\alpha_1=0, \alpha_3\neq 0, \alpha_5=0$, then by choosing $r=\alpha_3, x=\alpha_5$, we have the family of representatives $\langle \nabla_3+\alpha\nabla_4+\nabla_7\rangle$;

(f) if $\alpha_1=0, \alpha_3\neq 0, \alpha_5\neq 0$, then by choosing $x=\alpha_3{\alpha_5}^{-1}, r=\alpha_3^2\alpha_5^{-1}\alpha_7^{-1}$, we have the family of representatives $\langle \nabla_3+\alpha\nabla_4+\nabla_5+\nabla_7\rangle$;

(g) if $\alpha_1\neq 0, \alpha_3=0, \alpha_4=0, \alpha_5=0$, then by choosing $x=\alpha_1\alpha_7, r=\alpha_1$, we have the representative $\langle \nabla_1+\nabla_7\rangle$;

(h) if $\alpha_1\neq 0, \alpha_3=0, \alpha_4=0, \alpha_5\neq 0$, then by choosing $x=\sqrt[3]{\alpha_1\alpha_7\alpha_5^{-2}}, r=\sqrt[3]{\alpha_1^2\alpha_5^{-1}\alpha_7^{-1}}$, we have the representative $\langle \nabla_1+\nabla_5+\nabla_7\rangle$;

(i) if $\alpha_1\neq 0, \alpha_3=0, \alpha_4\neq 0$, then by choosing $x=\alpha_1{\alpha_4}^{-1}, r=\alpha_1\sqrt{\alpha_4^{-1}\alpha_7^{-1}}$, we have the family of representatives $\langle \nabla_1+\nabla_4+\alpha\nabla_5+\nabla_7\rangle$;

(j) if $\alpha_1 \neq 0, \alpha_3 \neq 0$, then by choosing $x = \alpha_1\alpha_7\alpha_3^{-2}, r = \alpha_1\alpha_3{}^{-1}$, we have the family of representatives $\langle\nabla_1 + \nabla_3 + \alpha\nabla_4 + \beta\nabla_5 + \nabla_7\rangle$.

4. $\alpha_8 \neq 0$, then by choosing $x = \alpha_8, t = -\alpha_3, s = -\alpha_7, r = \alpha_8$, we have $\alpha_3^* = \alpha_7^* = 0$. Now we can suppose that $\alpha_3 = 0$ and $\alpha_7 = 0$, then for $s = 0$ and $t = 0$, we have
   (a) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_6 = 0$, then we have the representatives $\langle\nabla_8\rangle$ and $\langle\nabla_5 + \nabla_8\rangle$, depending on whether $\alpha_5 = 0$ or not;
   (b) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_6 \neq 0$, then we have the representatives $\langle\nabla_6 + \nabla_8\rangle$ and $\langle\nabla_5 + \nabla_6 + \nabla_8\rangle$, depending on whether $\alpha_5 = 0$ or not;
   (c) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 \neq 0, \alpha_6 = 0$, then we have the representatives $\langle\nabla_4 + \nabla_8\rangle$ and $\langle\nabla_4 + \nabla_5 + \nabla_8\rangle$, depending on whether $\alpha_5 = 0$ or not;
   (d) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 \neq 0, \alpha_6 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_6\alpha_8{}^{-1}}, r = \sqrt{\alpha_4\alpha_8{}^{-1}}$, we have the representative $\langle\nabla_4 + \alpha\nabla_5 + \nabla_6 + \nabla_8\rangle$;
   (e) if $\alpha_1 = 0, \alpha_2 \neq 0, \alpha_4 = 0, \alpha_5 = 0$, then we have the representatives $\langle\nabla_2 + \nabla_8\rangle$ and $\langle\nabla_2 + \nabla_6 + \nabla_8\rangle$, depending on whether $\alpha_6 = 0$ or not;
   (f) if $\alpha_1 = 0, \alpha_2 \neq 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_2\alpha_5{}^{-1}}, r = \alpha_5\alpha_8{}^{-1}$, we have the representative $\langle\nabla_2 + \nabla_5 + \alpha\nabla_6 + \nabla_8\rangle$;
   (g) if $\alpha_1 = 0, \alpha_2 \neq 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[6]{\alpha_2^2\alpha_4^{-1}\alpha_8^{-1}}$, $r = \sqrt{\alpha_4\alpha_8{}^{-1}}$, we have the representative $\langle\nabla_2 + \nabla_4 + \alpha\nabla_5 + \beta\nabla_6 + \nabla_8\rangle$;
   (h) if $\alpha_1 \neq 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_5 = 0$, then we have the representatives $\langle\nabla_1 + \nabla_8\rangle$ and $\langle\nabla_1 + \nabla_6 + \nabla_8\rangle$ depending on whether $\alpha_6 = 0$ or not;
   (i) if $\alpha_1 \neq 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $x = \alpha_1\alpha_8\alpha_5^{-2}, r = \alpha_5\alpha_8{}^{-1}$, we have the representative $\langle\nabla_1 + \nabla_5 + \alpha\nabla_6 + \nabla_8\rangle$;
   (j) if $\alpha_1 \neq 0, \alpha_2 = 0, \alpha_4 \neq 0$, then by choosing $x = \alpha_1\alpha_4{}^{-1}, r = \sqrt{\alpha_4\alpha_8{}^{-1}}$, we have the representative $\langle\nabla_1 + \nabla_4 + \alpha\nabla_5 + \beta\nabla_6 + \nabla_8\rangle$;
   (k) if $\alpha_1 \neq 0, \alpha_2 \neq 0$, then by choosing $x = \sqrt[5]{\alpha_2^2\alpha_1^{-1}\alpha_8^{-1}}, r = \sqrt[5]{\alpha_1^4\alpha_2^{-3}\alpha_8^{-1}}$, we have the representative $\langle\nabla_1 + \nabla_2 + \alpha\nabla_4 + \beta\nabla_5 + \gamma\nabla_6 + \nabla_8\rangle$.

Summarizing, we have the following distinct orbits:

$$\langle\nabla_1 + \nabla_2 + \alpha\nabla_4 + \beta\nabla_5 + \gamma\nabla_6 + \nabla_8\rangle^{\substack{O(\alpha,\beta,\gamma) = O(-\eta_5\alpha, -\eta_5^3\beta, -\eta_5\gamma) = \\ O(\eta_5^2\alpha, -\eta_5\beta, \eta_5^2\gamma) = O(-\eta_5^3\alpha, \eta_5^4\beta, -\eta_5^3\gamma) = \\ O(\eta_5^4\alpha, \eta_5^2\beta, \eta_5^4\gamma)}},$$
$$\langle\nabla_1 + \nabla_3\rangle, \langle\nabla_1 + \nabla_3 + \alpha\nabla_4 + \beta\nabla_5 + \nabla_7\rangle, \langle\nabla_1 + \nabla_3 + \nabla_4 + \alpha\nabla_6\rangle,$$
$$\langle\nabla_1 + \nabla_3 + \nabla_6\rangle, \langle\nabla_1 + \nabla_4 + \alpha\nabla_5 + \beta\nabla_6 + \nabla_8\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)},$$
$$\langle\nabla_1 + \nabla_4 + \alpha\nabla_5 + \nabla_7\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)}, \langle\nabla_1 + \nabla_5 + \alpha\nabla_6 + \nabla_8\rangle, \langle\nabla_1 + \nabla_5 + \nabla_7\rangle,$$
$$\langle\nabla_1 + \nabla_6 + \nabla_8\rangle, \langle\nabla_1 + \nabla_7\rangle, \langle\nabla_1 + \nabla_8\rangle,$$
$$\langle\nabla_2 + \nabla_4 + \nabla_5 + \alpha\nabla_6\rangle^{O(\alpha)=O(\eta_3\alpha)=O(\eta_3^2\alpha)}, \langle\nabla_2 + \nabla_4 + \alpha\nabla_5 + \beta\nabla_6 + \nabla_8\rangle,$$
$$\langle\nabla_2 + \nabla_5\rangle, \langle\nabla_2 + \nabla_5 + \nabla_6\rangle,$$
$$\langle\nabla_2 + \nabla_5 + \alpha\nabla_6 + \nabla_8\rangle^{\substack{O(\alpha,\beta) = O(-\alpha,\beta) = O(\alpha,\eta_3^2\beta) = O(-\alpha,\eta_3^2\beta) = \\ O(-\alpha,-\eta_3\beta) = O(\alpha,-\eta_3\beta)}},$$
$$\langle\nabla_2 + \nabla_6 + \nabla_8\rangle, \langle\nabla_2 + \nabla_8\rangle, \langle\nabla_3\rangle, \langle\nabla_3 + \nabla_4\rangle, \langle\nabla_3 + \nabla_4 + \nabla_5 + \alpha\nabla_6\rangle,$$
$$\langle\nabla_3 + \alpha\nabla_4 + \nabla_5 + \nabla_7\rangle, \langle\nabla_3 + \nabla_4 + \nabla_6\rangle, \langle\nabla_3 + \nabla_5\rangle, \langle\nabla_3 + \nabla_5 + \nabla_6\rangle,$$
$$\langle\nabla_3 + \nabla_6\rangle,$$
$$\langle\nabla_4 + \nabla_5\rangle, \langle\nabla_4 + \nabla_5 + \nabla_6\rangle, \langle\nabla_4 + \alpha\nabla_5 + \nabla_6 + \nabla_8\rangle^{O(\alpha)=O(-\alpha)},$$
$$\langle\nabla_4 + \nabla_5 + \nabla_7\rangle, \langle\nabla_4 + \nabla_5 + \nabla_8\rangle, \langle\nabla_4 + \nabla_7\rangle, \langle\nabla_4 + \nabla_8\rangle, \langle\nabla_5\rangle, \langle\nabla_5 + \nabla_6\rangle,$$
$$\langle\nabla_5 + \nabla_6 + \nabla_8\rangle, \langle\nabla_5 + \nabla_7\rangle, \langle\nabla_5 + \nabla_8\rangle, \langle\nabla_6 + \nabla_8\rangle, \langle\nabla_7\rangle, \langle\nabla_8\rangle,$$

which gives the following new algebras:

$$\mathbf{N}_{83}^{\alpha,\beta,\gamma} : \quad e_1e_1 = e_2 \quad e_1e_2 = e_5 \quad e_1e_3 = e_5 \quad e_2e_2 = \alpha e_5$$

		$e_2e_3 = e_4$	$e_2e_4 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{84}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	
$\mathbf{N}_{85}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_4 = e_5$	$e_2e_2 = \alpha e_5$	
		$e_2e_3 = e_4$	$e_2e_4 = \beta e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{86}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_4 = e_5$		
		$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{87}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{88}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	
		$e_2e_4 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{89}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_2 = e_5$		
		$e_2e_3 = e_4$	$e_2e_4 = \alpha e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{90}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$		
		$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{91}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{92}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{93}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{94}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	$e_4e_4 = e_5$	
$\mathbf{N}_{95}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_2 = e_5$		
		$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{96}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_2 = e_5$		
		$e_2e_3 = e_4$	$e_2e_4 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{97}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	
$\mathbf{N}_{98}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{99}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$		
		$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{100}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{101}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	$e_4e_4 = e_5$	
$\mathbf{N}_{102}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$		
$\mathbf{N}_{103}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	
$\mathbf{N}_{104}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_5$		
		$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{105}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = \alpha e_5$		
		$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{106}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{107}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	
$\mathbf{N}_{108}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{109}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	
$\mathbf{N}_{110}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	
$\mathbf{N}_{111}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{112}^{\alpha}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$		
		$e_2e_4 = \alpha e_5$	$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{113}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{114}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{115}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{116}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_4e_4 = e_5$	
$\mathbf{N}_{117}$	:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$		
$\mathbf{N}_{118}$	:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{119}$	:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{120}$	:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_4 = e_5$	

$\mathbf{N}_{121}$	:	$e_1e_1=e_2$	$e_2e_3=e_4$	$e_2e_4=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{122}$	:	$e_1e_1=e_2$	$e_2e_3=e_4$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{123}$	:	$e_1e_1=e_2$	$e_2e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{124}$	:	$e_1e_1=e_2$	$e_2e_3=e_4$	$e_4e_4=e_5$	

3.5. **1-dimensional central extensions of $\mathbf{N}_{10}^{4*}$.** Here we will collect all information about $\mathbf{N}_{10}^{4*}$:

$\mathbf{N}_{10}^{4*}$	$e_1e_1=e_2$ $e_1e_2=e_4$ $e_3e_3=e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{10}^{4*})=\Big\langle [\Delta_{13}],[\Delta_{14}],[\Delta_{22}], [\Delta_{23}],[\Delta_{33}]\Big\rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{10}^{4*})=\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{10}^{4*})\oplus \langle[\Delta_{24}],[\Delta_{34}],[\Delta_{44}]\rangle$	$\phi=\begin{pmatrix} x & 0 & 0 & 0\\ y & x^2 & -\frac{zr}{x} & 0\\ z & 0 & r & 0\\ t & z^2+2xy & s & x^3\end{pmatrix},$ $r^2=x^3$

Let us use the following notations:

$$\begin{array}{llll}\nabla_1=[\Delta_{13}], & \nabla_2=[\Delta_{14}], & \nabla_3=[\Delta_{22}], & \nabla_4=[\Delta_{23}],\\ \nabla_5=[\Delta_{24}], & \nabla_6=[\Delta_{33}], & \nabla_7=[\Delta_{34}], & \nabla_8=[\Delta_{44}].\end{array}$$

Take $\theta=\sum\limits_{i=1}^{8}\alpha_i\nabla_i\in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{10}^{4*})$. Since

$$\phi^T\begin{pmatrix} 0 & 0 & \alpha_1 & \alpha_2\\ 0 & \alpha_3 & \alpha_4 & \alpha_5\\ \alpha_1 & \alpha_4 & \alpha_6 & \alpha_7\\ \alpha_2 & \alpha_5 & \alpha_7 & \alpha_8\end{pmatrix}\phi=\begin{pmatrix} \alpha^* & \alpha^{**} & \alpha_1^* & \alpha_2^*\\ \alpha^{**} & \alpha_3^* & \alpha_4^* & \alpha_5^*\\ \alpha_1^* & \alpha_4^* & \alpha_6^*+\alpha^{**} & \alpha_7^*\\ \alpha_2^* & \alpha_5^* & \alpha_7^* & \alpha_8^*\end{pmatrix},$$

we have

$$\begin{array}{rcl}
\alpha_1^* &=& -(\alpha_3y+\alpha_4z+\alpha_5t)\frac{zr}{x}+(\alpha_1x+\alpha_4y+\alpha_6z+\alpha_7t)r+\\
&& (\alpha_2x+\alpha_5y+\alpha_7z+\alpha_8t)s,\\
\alpha_2^* &=& (\alpha_2x+\alpha_5y+\alpha_7z+\alpha_8t)x^3,\\
\alpha_3^* &=& \alpha_3x^4+2\alpha_5x^2(z^2+2xy)+\alpha_8(z^2+2xy)^2,\\
\alpha_4^* &=& -(\alpha_3x^2+\alpha_5(z^2+2xy))\frac{zr}{x}+(\alpha_4x^2+\alpha_7(z^2+2xy))r+\\
&& (\alpha_5x^2+\alpha_8(z^2+2xy))s,\\
\alpha_5^* &=& (\alpha_5x^2+\alpha_8(z^2+2xy))x^3,\\
\alpha_6^* &=& -(\alpha_4r-\alpha_3\frac{zr}{x}+\alpha_5s)\frac{zr}{x}+(\alpha_6r-\alpha_4\frac{zr}{x}+\alpha_7s)r+\\
&& (\alpha_7r-\alpha_5\frac{zr}{x}+\alpha_8s)s-(\alpha_3y+\alpha_4z+\alpha_5t)x^2-\\
&& (\alpha_2x+\alpha_5y+\alpha_7z+\alpha_8t)(z^2+2xy),\\
\alpha_7^* &=& (\alpha_7r-\alpha_5\frac{zr}{x}+\alpha_8s)x^3,\\
\alpha_8^* &=& \alpha_8x^6.
\end{array}$$

We are interested in $(\alpha_5,\alpha_7,\alpha_8)\neq(0,0,0)$. Let us consider the following cases:

1. $\alpha_8=0,\alpha_5=0$, then $\alpha_7\neq 0$. Now by choosing
$$y=-\frac{\alpha_2^2+\alpha_2\alpha_3+\alpha_4\alpha_7}{2\alpha_7^2}x,\ z=-\frac{\alpha_2}{\alpha_7}x,$$
$$s=-\frac{3\alpha_2^2\alpha_3+\alpha_2(\alpha_3^2+6\alpha_4\alpha_7)+\alpha_7(\alpha_3\alpha_4+2\alpha_6\alpha_7)}{4\alpha_7^3}\sqrt{x^3},$$
$$t=\frac{\alpha_7^2(\alpha_4^2-2\alpha_1\alpha_7)+\alpha_2^3\alpha_3+\alpha_2^2(\alpha_3^2+3\alpha_4\alpha_7)+2\alpha_2\alpha_7(\alpha_3\alpha_4+\alpha_6\alpha_7)}{2\alpha_7^4}x,$$
we have $\alpha_1^*=0,\alpha_2^*=0,\alpha_4^*=0,\alpha_6^*=0$. Then we have the representatives $\langle\nabla_7\rangle$ or $\langle\nabla_3+\nabla_7\rangle$ depending on whether $\alpha_3=0$ or not.

2. $\alpha_8=0,\alpha_5\neq 0$, then by choosing
$$y=-\frac{\alpha_2\alpha_5+\alpha_7^2}{\alpha_5^2}x,\ z=\frac{\alpha_7}{\alpha_5}x,\ s=\frac{\alpha_3\alpha_7-\alpha_4\alpha_5}{\alpha_5^2}\sqrt{x^3},$$
$$t=\frac{\alpha_2\alpha_3\alpha_5+\alpha_5^2\alpha_6+3\alpha_4\alpha_5\alpha_7-2\alpha_3\alpha_7^2}{\alpha_5^3}x,$$
we have $\alpha_2^*=\alpha_4^*=\alpha_6^*=0$ and $\alpha_7^*=0$. Therefore, we can suppose that $\alpha_2=0,\alpha_4=0,\alpha_6=0,\alpha_7=0$, and have the following cases:
   (a) if $\alpha_1=0,\alpha_3=0$, then we have the representative $\langle\nabla_5\rangle$;
   (b) if $\alpha_1=0,\alpha_3\neq 0$, then by choosing $x=\frac{\alpha_3}{\alpha_5}, r^2=x^3$, we have the representative $\langle\nabla_3+\nabla_5\rangle$;
   (c) if $\alpha_1\neq 0$, then by choosing $x=\sqrt[5]{\frac{\alpha_1^2}{\alpha_5^2}}, r^2=x^3$, we have the family of representatives $\langle\nabla_1+\alpha\nabla_3+\nabla_5\rangle$.
3. $\alpha_8\neq 0$, then by choosing $y=-\frac{\alpha_5x^2+\alpha_8z^2}{2\alpha_8x}, s=\frac{\sqrt{x}(\alpha_5z-\alpha_7x)}{\alpha_8}, t=-\frac{\alpha_2x+\alpha_5y+\alpha_7z}{\alpha_8}$, we have $\alpha_2^*=\alpha_5^*=\alpha_7^*=0$. Therefore, we can suppose that $\alpha_2=0,\alpha_5=0,\alpha_7=0$, and have the following cases:
   (a) if $\alpha_3=0,\alpha_4=0,\alpha_6=0$, then we have the representative $\langle\nabla_8\rangle$ and $\langle\nabla_1+\nabla_8\rangle$ depending on whether $\alpha_1=0$ or not.
   (b) if $\alpha_3=0,\alpha_4=0,\alpha_6\neq 0$, then by choosing $x=\sqrt[3]{\frac{\alpha_6}{\alpha_8}}, r^2=x^3, z=-\frac{\alpha_1}{\sqrt[3]{\alpha_6^2\alpha_8}}$, we have the representative $\langle\nabla_6+\nabla_8\rangle$;
   (c) if $\alpha_3=0,\alpha_4\neq 0$, then by choosing $x=\sqrt[5]{\frac{\alpha_4^2}{\alpha_8^2}}, r^2=x^3, z=\frac{\alpha_6}{3\sqrt[5]{\alpha_4^3\alpha_8^2}}$, we have the representative $\langle\alpha\nabla_1+\nabla_4+\nabla_8\rangle$;
   (d) if $\alpha_3\neq 0$, then by choosing $x=\sqrt{\frac{\alpha_3}{\alpha_8}}, r^2=x^3, z=\frac{\alpha_4}{\sqrt{\alpha_3\alpha_8}}$, we have the representative $\langle\alpha\nabla_1+\nabla_3+\beta\nabla_6+\nabla_8\rangle$.

Summarizing, we have the following distinct orbits:
$$\langle\nabla_1+\alpha\nabla_3+\nabla_5\rangle^{O(\alpha)=O(\eta_5^4\alpha)=O(-\eta_5^3\alpha)=O(\eta_5^2\alpha)=O(-\eta_5\alpha)},$$
$$\langle\alpha\nabla_1+\nabla_3+\beta\nabla_6+\nabla_8\rangle^{\substack{O(\alpha,\beta)=O(-\alpha,\beta)=O(\eta_3\alpha,\eta_3^2\beta)=\\ O(-\eta_3\alpha,\eta_3^2\beta)=O(-\eta_3^2\alpha,-\eta_3\beta)=O(\eta_3^2\alpha,-\eta_3\beta)}},$$
$$\langle\alpha\nabla_1+\nabla_4+\nabla_8\rangle^{\substack{O(\alpha)=O(-\alpha)=O(\eta_5^4\alpha)=O(-\eta_5^4\alpha)=O(\eta_5^3\alpha)=\\ O(-\eta_5^3\alpha)=O(\eta_5^2\alpha)=O(-\eta_5^2\alpha)=O(\eta_5\alpha)=O(-\eta_5\alpha)}},$$
$$\langle\nabla_1+\nabla_8\rangle, \langle\nabla_3+\nabla_5\rangle, \langle\nabla_3+\nabla_7\rangle, \langle\nabla_5\rangle, \langle\nabla_6+\nabla_8\rangle, \langle\nabla_7\rangle, \langle\nabla_8\rangle,$$
which gives the following new algebras:

$\mathbf{N}_{125}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_5$		
		$e_2e_2=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{126}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=\alpha e_5$		
		$e_2e_2=e_5$	$e_3e_3=e_4+\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{127}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=\alpha e_5$		
		$e_2e_3=e_5$	$e_3e_3=e_4$	$e_4e_4=e_5$		
$\mathbf{N}_{128}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_5$	$e_3e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{129}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_2=e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{130}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_2=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{131}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{132}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_3e_3=e_4+e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{133}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{134}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_3e_3=e_4$	$e_4e_4=e_5$	

3.6. **1-dimensional central extensions of $\mathbf{N}_{11}^{4*}$.** Here we will collect all information about $\mathbf{N}_{11}^{4*}$:

$\mathbf{N}_{11}^{4*}$	$e_1e_1 = e_2$ $e_1e_3 = e_4$ $e_2e_2 = e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{11}^{4*}) = \langle [\Delta_{12}], [\Delta_{22}], [\Delta_{23}], [\Delta_{33}] \rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{11}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{11}^{4*}) \oplus \langle [\Delta_{14}], [\Delta_{24}], [\Delta_{34}], [\Delta_{44}] \rangle$	$\phi = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & x^2 & 0 & 0 \\ z & 0 & x^3 & 0 \\ t & 2xz & s & x^4 \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{12}], & \nabla_2 = [\Delta_{14}], & \nabla_3 = [\Delta_{22}], & \nabla_4 = [\Delta_{23}], \\ \nabla_5 = [\Delta_{24}], & \nabla_6 = [\Delta_{33}], & \nabla_7 = [\Delta_{34}], & \nabla_8 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{11}^{4*})$. Since

$$\phi^T \begin{pmatrix} 0 & \alpha_1 & 0 & \alpha_2 \\ \alpha_1 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & \alpha_4 & \alpha_6 & \alpha_7 \\ \alpha_2 & \alpha_5 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha_1^* & \alpha^{**} & \alpha_2^* \\ \alpha_1^* & \alpha_3^* + \alpha^{**} & \alpha_4^* & \alpha_5^* \\ \alpha^{**} & \alpha_4^* & \alpha_6^* & \alpha_7^* \\ \alpha_2^* & \alpha_5^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

we have

$$\begin{array}{lll} \alpha_1^* &=& (\alpha_1 x + \alpha_4 z + \alpha_5 t)x^2 + 2(\alpha_2 x + \alpha_7 z + \alpha_8 t)xz, \\ \alpha_2^* &=& (\alpha_2 x + \alpha_7 z + \alpha_8 t)x^4, \\ \alpha_3^* &=& (\alpha_3 x^2 + 4\alpha_5 xz + 4\alpha_8 z^2)x^2 - (\alpha_6 z + \alpha_7 t)x^3 - (\alpha_2 x + \alpha_7 z + \alpha_8 t)s, \\ \alpha_4^* &=& (\alpha_4 x + 2\alpha_7 z)x^4 + (\alpha_5 x + 2\alpha_8 z)xs, \\ \alpha_5^* &=& (\alpha_5 x + 2\alpha_8 z)x^5, \\ \alpha_6^* &=& \alpha_6 x^6 + 2\alpha_7 x^3 s + \alpha_8 s^2, \\ \alpha_7^* &=& (\alpha_7 x^3 + \alpha_8 s)x^4, \\ \alpha_8^* &=& \alpha_8 x^8. \end{array}$$

We are interested in $(\alpha_2, \alpha_5, \alpha_7, \alpha_8) \neq (0,0,0,0)$. Let us consider the following cases:

1. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 = 0$, then $\alpha_2 \neq 0$ and we have
   (a) if $\alpha_4 = 0, \alpha_6 = 0$, then by choosing $x = 2\alpha_2, z = -\alpha_1, s = 8\alpha_2^2\alpha_3, t = 0$, we have the representative $\langle \nabla_2 \rangle$;
   (b) if $\alpha_4 = 0, \alpha_6 \neq 0$, then by choosing $x = \frac{\alpha_2}{\alpha_6}, z = -\frac{\alpha_1}{2\alpha_6}, s = \frac{\alpha_1\alpha_2\alpha_6 + 2\alpha_2^2\alpha_3}{2\alpha_6^3}, t = 0$, we have the representative $\langle \nabla_2 + \nabla_6 \rangle$;
   (c) if $\alpha_4 \neq 0, \alpha_4 = -2\alpha_2, \alpha_1 \neq 0$, then by choosing
   $$x = \sqrt{\frac{\alpha_1}{\alpha_2}}, z = 0, s = \frac{\alpha_1\alpha_3\sqrt{\alpha_1}}{\alpha_2^2\sqrt{\alpha_2}}, t = 0,$$
   we have the family of representatives $\langle \nabla_1 + \nabla_2 - 2\nabla_4 + \alpha\nabla_6 \rangle$;
   (d) if $\alpha_4 \neq 0, \alpha_4 = -2\alpha_2, \alpha_1 = 0, \alpha_6 \neq 0$, then by choosing $x = \alpha_2\alpha_6^{-1}$, $z = 0$, $s = \alpha_2^2\alpha_3\alpha_6^{-3}$, $t = 0$, we have the representative $\langle \nabla_2 - 2\nabla_4 + \nabla_6 \rangle$;
   (e) if $\alpha_4 \neq 0, \alpha_4 = -2\alpha_2, \alpha_1 = 0, \alpha_6 = 0$, then by choosing $x = \alpha_2$, $z = 0$, $s = \alpha_2^2\alpha_3$, $t = 0$, we have the representative $\langle \nabla_2 - 2\nabla_4 \rangle$;
   (f) if $\alpha_4 \neq 0, \alpha_4 \neq -2\alpha_2, \alpha_6 = 0$, then by choosing
   $$x = \alpha_4 + 2\alpha_2, z = -\alpha_1, s = \frac{\alpha_3(\alpha_4 + 2\alpha_2)^3}{\alpha_2}, t = 0,$$
   we have the the family of representatives $\langle \nabla_2 + \alpha\nabla_4 \rangle_{\alpha \neq 0, -2}$, which will be jointed with the cases (1a) and (1e);

(g) if $\alpha_4 \neq 0, \alpha_4 \neq -2\alpha_2, \alpha_6 \neq 0$, then by choosing
$$x = \frac{\alpha_2}{\alpha_6}, z = -\frac{\alpha_1\alpha_2}{\alpha_6(\alpha_4+2\alpha_2)}, s = \frac{\alpha_1\alpha_2^2\alpha_6+2\alpha_2^3\alpha_3+\alpha_2^2\alpha_3\alpha_4}{\alpha_6^3(\alpha_4+2\alpha_2)}, t = 0,$$
we have the family of representatives $\langle \nabla_2 + \alpha\nabla_4 + \nabla_6 \rangle_{\alpha\neq 0,-2}$, which will be jointed with the cases (1b) and (1d).

2. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 \neq 0$, then we have
(a) if $\alpha_6 = 0, \alpha_2 = 0$, then by choosing
$$x = 4\alpha_5, z = -\alpha_3, s = -64\alpha_4\alpha_5^2, t = \frac{\alpha_3\alpha_4-4\alpha_1\alpha_5}{\alpha_5}$$
we have the representative $\langle \nabla_5 \rangle$;
(b) if $\alpha_6 = 0, \alpha_2 \neq 0$, then by choosing
$$x = \frac{\alpha_2}{\alpha_5},\ z = -\frac{\alpha_2^2\alpha_4+\alpha_2\alpha_3\alpha_5}{4\alpha_5^3},\ s = -\frac{\alpha_2^3\alpha_4}{\alpha_5^4},$$
$$t = \frac{(\alpha_2\alpha_4+2\alpha_2^2)(\alpha_2\alpha_4+\alpha_3\alpha_5)-4\alpha_1\alpha_2\alpha_5^2}{4\alpha_5^4},$$
we have the representative $\langle \nabla_2 + \nabla_5 \rangle$;
(c) if $\alpha_6 \neq 0, \alpha_6 = 4\alpha_5, \alpha_3 = 0, \alpha_2 = 0$, then by choosing $x = \alpha_5, z = 0, s = -\alpha_4\alpha_5^2, t = -\alpha_1$, we have the representative $\langle \nabla_5 + 4\nabla_6 \rangle$;
(d) if $\alpha_6 \neq 0, \alpha_6 = 4\alpha_5, \alpha_2\alpha_4 + \alpha_3\alpha_5 = 0, \alpha_2 \neq 0$, then by choosing
$$x = \frac{\alpha_2}{\alpha_5}, z = 0, s = -\frac{\alpha_2^3\alpha_4}{\alpha_5^4}, t = -\frac{\alpha_1\alpha_2}{\alpha_5^2},$$
we have the representative $\langle \nabla_2 + \nabla_5 + 4\nabla_6 \rangle$;
(e) if $\alpha_6 \neq 0, \alpha_6 = 4\alpha_5, \alpha_2\alpha_4 + \alpha_3\alpha_5 \neq 0$, then by choosing
$$x = \frac{\sqrt{\alpha_2\alpha_4+\alpha_3\alpha_5}}{\alpha_5}, z = 0,\ s = -\frac{\alpha_4\sqrt{(\alpha_2\alpha_4+\alpha_3\alpha_5)^3}}{\alpha_5^4},\ t = -\frac{\alpha_1\sqrt{\alpha_2\alpha_4+\alpha_3\alpha_5}}{\alpha_5^2},$$
we have the family of representatives $\langle \alpha\nabla_2 + \nabla_3 + \nabla_5 + 4\nabla_6 \rangle$;
(f) if $\alpha_6 \neq 0, \alpha_6 \neq 4\alpha_5, \alpha_2 = 0$, then by choosing
$$x = \alpha_6 - 4\alpha_5, z = \alpha_3,\ s = \frac{\alpha_4(4\alpha_5-\alpha_6)^3}{\alpha_5},\ t = \frac{4\alpha_1\alpha_5-\alpha_1\alpha_6-\alpha_3\alpha_4}{\alpha_5},$$
we have the family of representatives $\langle \nabla_5 + \alpha\nabla_6 \rangle_{\alpha\neq 0,4}$, which will be jointed with the cases (2a) and (2c);
(g) if $\alpha_6 \neq 0, \alpha_6 \neq 4\alpha_5, \alpha_2 \neq 0$, then by choosing
$$x = \frac{\alpha_2}{\alpha_5}, z = \frac{\alpha_2(\alpha_2\alpha_4+\alpha_3\alpha_5)}{\alpha_5^2\alpha_6-4\alpha_5^2},\ s = -\frac{\alpha_4\alpha_2^3}{\alpha_5^4},$$
$$t = \frac{\alpha_2(2\alpha_2^2\alpha_4+\alpha_3\alpha_4\alpha_5+\alpha_2(\alpha_4^2+2\alpha_3\alpha_5)-\alpha_1\alpha_5(4\alpha_5-\alpha_6))}{\alpha_5^3(4\alpha_5-\alpha_6)},$$
we have the family of representatives $\langle \nabla_2 + \nabla_5 + \alpha\nabla_6 \rangle_{(\alpha\neq 0,4)}$, which will be jointed with the cases (2b) and (2d).

3. $\alpha_8 = 0, \alpha_7 \neq 0$, then by choosing $z = -\frac{\alpha_2}{\alpha_7}x,\ s = -\frac{\alpha_6}{2\alpha_7}x^3,\ t = \frac{\alpha_2(\alpha_6-4\alpha_5)+\alpha_3\alpha_7}{\alpha_7^2}x$, we have $\alpha_2^* = \alpha_3^* = \alpha_6^* = 0$. Now we can suppose that $\alpha_2 = 0, \alpha_3 = 0, \alpha_6 = 0$ and have the following subcases:
(a) if $\alpha_1 = 0, \alpha_4 = 0, \alpha_5 = 0$, then we have the representative $\langle \nabla_7 \rangle$;
(b) if $\alpha_1 = 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $x = \frac{\alpha_5}{\alpha_7}, z = 0, s = 0, t = 0$, we have the representative $\langle \nabla_5 + \nabla_7 \rangle$;
(c) if $\alpha_1 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_4}{\alpha_7}}, z = 0, s = 0, t = 0$, we have the family of representatives $\langle \nabla_4 + \alpha\nabla_5 + \nabla_7 \rangle$;
(d) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_1}{\alpha_7}}, z = 0, s = 0, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha\nabla_4 + \beta\nabla_5 + \nabla_7 \rangle$.

4. $\alpha_8 \neq 0$, then by choosing $z = -\frac{\alpha_5}{2\alpha_8}x, s = -\frac{\alpha_7}{\alpha_8}x^3, t = \frac{\alpha_5\alpha_7-2\alpha_2\alpha_8}{2\alpha_8^2}x$, we have $\alpha_2^* = \alpha_5^* = \alpha_7^* = 0$. Now we can suppose that $\alpha_2 = 0, \alpha_5 = 0, \alpha_7 = 0$ and have the following subcases:
(a) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_6 = 0$, then we have the representative $\langle \nabla_8 \rangle$;

(b) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_6 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_6}{\alpha_8}}, z = 0, s = 0, t = 0$, we have the representative $\langle \nabla_6 + \nabla_8 \rangle$;

(c) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[3]{\frac{\alpha_4}{\alpha_8}}, z = 0, s = 0, t = 0$, we have the family of representatives $\langle \nabla_4 + \alpha\nabla_6 + \nabla_8 \rangle$;

(d) if $\alpha_1 = 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_3}{\alpha_8}}, z = 0, s = 0, t = 0$, we have the family of representatives $\langle \nabla_3 + \alpha\nabla_4 + \beta\nabla_6 + \nabla_8 \rangle$;

(e) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[5]{\frac{\alpha_1}{\alpha_8}}, z = 0, s = 0, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \gamma\nabla_6 + \nabla_8 \rangle$.

Summarizing, we have the following distinct orbits:

$$\langle \nabla_1 + \nabla_2 - 2\nabla_4 + \alpha\nabla_6 \rangle^{O(\alpha)=O(-\alpha)},$$
$$\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \gamma\nabla_6 + \nabla_8 \rangle^{\substack{O(\alpha,\beta,\gamma)=O(-\eta_5\alpha,\eta_5^2\beta,-\eta_5^3\gamma)=O(\eta_5^2\alpha,\eta_5^4\beta,-\eta_5\gamma)= \\ O(-\eta_5^3\alpha,-\eta_5\beta,\eta_5^4\gamma)=O(\eta_5^4\alpha,-\eta_5^3\beta,\eta_5^2\gamma)}},$$
$$\langle \nabla_1 + \alpha\nabla_4 + \beta\nabla_5 + \nabla_7 \rangle^{O(\alpha,\beta)=O(\alpha,-\beta)=O(-\alpha,-i\beta)=O(-\alpha,i\beta)},$$
$$\langle \alpha\nabla_2 + \nabla_3 + \nabla_5 + 4\nabla_6 \rangle^{O(\alpha)=O(-\alpha)}, \langle \nabla_2 + \alpha\nabla_4 \rangle, \langle \nabla_2 + \alpha\nabla_4 + \nabla_6 \rangle,$$
$$\langle \nabla_2 + \nabla_5 + \alpha\nabla_6 \rangle, \langle \nabla_3 + \alpha\nabla_4 + \beta\nabla_6 + \nabla_8 \rangle, \langle \nabla_4 + \alpha\nabla_5 + \nabla_7 \rangle,$$
$$\langle \nabla_4 + \alpha\nabla_6 + \nabla_8 \rangle, \langle \nabla_5 + \alpha\nabla_6 \rangle, \langle \nabla_5 + \nabla_7 \rangle, \langle \nabla_6 + \nabla_8 \rangle, \langle \nabla_7 \rangle, \langle \nabla_8 \rangle,$$

which gives the following new algebras:

$\mathbf{N}_{135}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = e_4$	$e_1e_4 = e_5$
		$e_2e_2 = e_4$	$e_2e_3 = -2e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{136}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = e_4$	$e_2e_2 = e_4 + \alpha e_5$
		$e_2e_3 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{137}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = e_4$	$e_2e_2 = e_4$
		$e_2e_3 = \alpha e_5$	$e_2e_4 = \beta e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{138}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	
		$e_2e_2 = e_4 + e_5$	$e_2e_4 = e_5$	$e_3e_3 = 4e_5$	
$\mathbf{N}_{139}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_4$	$e_2e_3 = \alpha e_5$		
$\mathbf{N}_{140}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_4$	$e_2e_3 = \alpha e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{141}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{142}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4 + e_5$	
		$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{143}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_2e_3 = e_5$	$e_2e_4 = \alpha e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{144}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_2e_3 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{145}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{146}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_2e_4 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{147}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{148}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{149}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_4e_4 = e_5$

3.7. **1-dimensional central extensions of $\mathbf{N}_{12}^{4*}$.** Here we will collect all information about $\mathbf{N}_{12}^{4*}$:

$\mathbf{N}_{12}^{4*}$	$e_1e_1=e_2$ $e_2e_2=e_4$ $e_3e_3=e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{12}^{4*}) = \langle[\Delta_{12}],[\Delta_{13}],[\Delta_{23}],[\Delta_{33}]\rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{12}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{12}^{4*})\oplus \langle[\Delta_{14}],[\Delta_{24}],[\Delta_{34}],[\Delta_{44}]\rangle$	$\phi_\pm=\begin{pmatrix} x & 0 & 0 & 0\\ 0 & x^2 & 0 & 0\\ 0 & 0 & \pm x^2 & 0\\ t & 0 & s & x^4\end{pmatrix}$

Let us use the following notations:

$$\nabla_1=[\Delta_{12}],\quad \nabla_2=[\Delta_{13}],\quad \nabla_3=[\Delta_{14}],\quad \nabla_4=[\Delta_{23}],$$
$$\nabla_5=[\Delta_{24}],\quad \nabla_6=[\Delta_{33}],\quad \nabla_7=[\Delta_{34}],\quad \nabla_8=[\Delta_{44}].$$

Take $\theta=\sum\limits_{i=1}^{8}\alpha_i\nabla_i\in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{12}^{4*})$. Since

$$\phi_\pm^T\begin{pmatrix} 0 & \alpha_1 & \alpha_2 & \alpha_3\\ \alpha_1 & 0 & \alpha_4 & \alpha_5\\ \alpha_2 & \alpha_4 & \alpha_6 & \alpha_7\\ \alpha_3 & \alpha_5 & \alpha_7 & \alpha_8\end{pmatrix}\phi_\pm=\begin{pmatrix} \alpha^* & \alpha_1^* & \alpha_2^* & \alpha_3^*\\ \alpha_1^* & 0 & \alpha_4^* & \alpha_5^*\\ \alpha_2^* & \alpha_4^* & \alpha_6^* & \alpha_7^*\\ \alpha_3^* & \alpha_5^* & \alpha_7^* & \alpha_8^*\end{pmatrix},$$

we have

$$\begin{array}{ll} \alpha_1^*=(\alpha_1x+\alpha_5t)x^2, & \alpha_2^*=(\alpha_3x+\alpha_8t)s\pm(\alpha_2x+\alpha_7t)x^2,\\ \alpha_3^*=(\alpha_3x+\alpha_8t)x^4, & \alpha_4^*=(\alpha_5s\pm\alpha_4x^2)x^2,\\ \alpha_5^*=\alpha_5x^6, & \alpha_6^*=\alpha_6x^4\pm2\alpha_7sx^2+\alpha_8s^2,\\ \alpha_7^*=(\alpha_8s\pm\alpha_7x^2)x^4, & \alpha_8^*=\alpha_8x^8. \end{array}$$

We will consider only the action of ϕ_+ for find representatives and after that we will see that the set of our representatives gives distinct orbits under action of ϕ_+ and ϕ_-. We are interested in $(\alpha_3,\alpha_5,\alpha_7,\alpha_8)\neq(0,0,0,0)$. Let us consider the following cases:

1. $\alpha_8=0,\alpha_5=0,\alpha_7=0$, then $\alpha_3\neq0$ and we have the following subcases:
   (a) if $\alpha_1=0,\alpha_4=0,\alpha_6=0$, then by choosing $x=\alpha_3, s=-\alpha_2\alpha_3, t=0$, we have the representative $\langle\nabla_3\rangle$;
   (b) if $\alpha_1=0,\alpha_4=0,\alpha_6\neq0$, then by choosing $x=\frac{\alpha_6}{\alpha_3}, s=-\frac{\alpha_2\alpha_6^2}{\alpha_3^3}, t=0$, we have the representative $\langle\nabla_3+\nabla_6\rangle$;
   (c) if $\alpha_1=0,\alpha_4\neq0$, then by choosing $x=\frac{\alpha_4}{\alpha_3}, s=-\frac{\alpha_2\alpha_4^2}{\alpha_3^3}, t=0$, we have the representative $\langle\nabla_3+\nabla_4+\alpha\nabla_6\rangle$;
   (d) if $\alpha_1\neq0$, then by choosing $x=\sqrt{\frac{\alpha_1}{\alpha_3}}, s=-\frac{\alpha_1\alpha_2}{\alpha_3^2}, t=0$, we have the representative $\langle\nabla_1+\nabla_3+\alpha\nabla_4+\beta\nabla_6\rangle$.
2. $\alpha_8=0,\alpha_5=0,\alpha_7\neq0$, then we have the following subcases:
   (a) if $\alpha_1=0,\alpha_3=0,\alpha_4=0$, then by choosing $x=1, s=-\frac{\alpha_6}{2\alpha_7}, t=\frac{\alpha_3\alpha_6-2\alpha_2\alpha_7}{2\alpha_7^2}$, we have the representative $\langle\nabla_7\rangle$;
   (b) if $\alpha_1=0,\alpha_3=0,\alpha_4\neq0$, then by choosing $x=\sqrt{\frac{\alpha_4}{\alpha_7}}, s=-\frac{\alpha_4\alpha_6}{2\alpha_7^2}, t=\frac{\sqrt{\alpha_4}(\alpha_3\alpha_6-2\alpha_2\alpha_4)}{2\alpha_7^2\sqrt{\alpha_7}}$, we have the representative $\langle\nabla_4+\nabla_7\rangle$;
   (c) if $\alpha_1=0,\alpha_3\neq0$, then by choosing $x=\frac{\alpha_3}{\alpha_7}, s=-\frac{\alpha_3^2\alpha_6}{2\alpha_7^3}, t=\frac{\alpha_3^2\alpha_6-2\alpha_2\alpha_3\alpha_7}{2\alpha_7^3}$, we have the representative $\langle\nabla_3+\alpha\nabla_4+\nabla_7\rangle$;

(d) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[3]{\frac{\alpha_1}{\alpha_7}}, s = -\sqrt[3]{\frac{\alpha_1^2\alpha_6^3}{8\alpha_7^5}}, t = \frac{\sqrt[3]{\alpha_1}(\alpha_3\alpha_6 - 2\alpha_2\alpha_7)}{2\alpha_7^2\sqrt[3]{\alpha_7}}$, we have the representative $\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \nabla_7 \rangle$.

3. $\alpha_8 = 0, \alpha_5 \neq 0$, then by choosing $t = -\frac{\alpha_1}{\alpha_5}x, s = -\frac{\alpha_4}{\alpha_5}x^2$, we have $\alpha_1^* = \alpha_4^* = 0$. Now we can suppose that $\alpha_1 = 0, \alpha_4 = 0$ and have the following subcases:
   (a) if $\alpha_2 = 0, \alpha_3 = 0, \alpha_6 = 0$, then we have the family of representatives $\langle \nabla_5 + \alpha\nabla_7 \rangle$;
   (b) if $\alpha_2 = 0, \alpha_3 = 0, \alpha_6 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_6}{\alpha_5}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_5 + \nabla_6 + \alpha\nabla_7 \rangle$;
   (c) if $\alpha_2 = 0, \alpha_3 \neq 0$, then by choosing $x = \frac{\alpha_3}{\alpha_5}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_3 + \nabla_5 + \alpha\nabla_6 + \beta\nabla_7 \rangle$;
   (d) if $\alpha_2 \neq 0$, then by choosing $x = \sqrt[3]{\frac{\alpha_2}{\alpha_5}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_2 + \alpha\nabla_3 + \nabla_5 + \beta\nabla_6 + \gamma\nabla_7 \rangle$.
4. $\alpha_8 \neq 0$, then by choosing $t = -\frac{\alpha_3}{\alpha_8}x, s = -\frac{\alpha_7}{\alpha_8}x^2$, we have $\alpha_3^* = \alpha_7^* = 0$. Now we can suppose that $\alpha_3 = 0, \alpha_7 = 0$ and have the following subcases:
   (a) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_5 = 0, \alpha_6 = 0$, then we have the representative $\langle \nabla_8 \rangle$;
   (b) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_5 = 0, \alpha_6 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_6}{\alpha_8}}, s = 0, t = 0$, we have the representative $\langle \nabla_6 + \nabla_8 \rangle$;
   (c) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_5}{\alpha_8}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_5 + \alpha\nabla_6 + \nabla_8 \rangle$;
   (d) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_4}{\alpha_8}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_4 + \alpha\nabla_5 + \beta\nabla_6 + \nabla_8 \rangle$;
   (e) if $\alpha_1 = 0, \alpha_2 \neq 0$, then by choosing $x = \sqrt[5]{\frac{\alpha_2}{\alpha_8}}, s = 0, t = 0$, we have the family of representatives
   $$\langle \nabla_2 + \alpha\nabla_4 + \beta\nabla_5 + \gamma\nabla_6 + \nabla_8 \rangle;$$
   (f) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[5]{\frac{\alpha_1}{\alpha_8}}, s = 0, t = 0$, we have the family of representatives
   $$\langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_4 + \gamma\nabla_5 + \mu\nabla_6 + \nabla_8 \rangle.$$

Summarizing all cases we have the following distinct orbits:

$$\langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_4 + \gamma\nabla_5 + \mu\nabla_6 + \nabla_8 \rangle^{\substack{O(\alpha,\beta,\gamma,\mu) = O(\pm\alpha, \pm\eta_5^4\beta, \eta_5^2\gamma, \eta_5^4\mu) = \\ O(\pm\alpha, \mp\eta_5^3\beta, \eta_5^4\gamma, -\eta_5^3\mu) = O(\pm\alpha, \pm\eta_5^2\beta, -\eta_5\gamma, \eta_5^2\mu) = \\ O(\pm\alpha, \mp\eta_5\beta, -\eta_5^3\gamma, -\eta_5\mu)}},$$
$$\langle \nabla_1 + \nabla_3 + \alpha\nabla_4 + \beta\nabla_6 \rangle^{\substack{O(\alpha,\beta) = O(-\alpha,\beta) = \\ O(\alpha,-\beta) = O(-\alpha,-\beta)}},$$
$$\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \nabla_7 \rangle^{\substack{O(\alpha,\beta) = O(-\eta_3\alpha, \eta_3^2\beta) = \\ O(\eta_3^2\alpha, -\eta_3\beta)}},$$
$$\langle \nabla_2 + \alpha\nabla_3 + \nabla_5 + \beta\nabla_6 + \gamma\nabla_7 \rangle^{\substack{O(\alpha,\beta,\gamma) = O(-\alpha,\beta,-\gamma) = O(-\eta_3\alpha, \eta_3^2\beta, \gamma) = \\ O(\eta_3\alpha, \eta_3^2\beta, -\gamma) = O(\eta_3^2\alpha, -\eta_3\beta, \gamma) = O(-\eta_3^2\alpha, -\eta_3\beta, -\gamma)}},$$
$$\langle \nabla_2 + \alpha\nabla_4 + \beta\nabla_5 + \gamma\nabla_6 + \nabla_8 \rangle^{\substack{O(\alpha,\beta,\gamma) = O(-\alpha,\beta,\gamma) = O(\eta_5^4\alpha, \eta_5^2\beta, \eta_5^4\gamma) = \\ O(-\eta_5^4\alpha, \eta_5^2\beta, \eta_5^4\gamma) = O(-\eta_5^3\alpha, \eta_5^4\beta, -\eta_5^3\gamma) = \\ O(\eta_5^3\alpha, \eta_5^4\beta, -\eta_5^3\gamma) = O(\eta_5^2\alpha, -\eta_5\beta, \eta_5^2\gamma) = \\ O(-\eta_5^2\alpha, -\eta_5\beta, \eta_5^2\gamma) = O(-\eta_5\alpha, -\eta_5^3\beta, -\eta_5\gamma) = \\ O(\eta_5\alpha, -\eta_5^3\beta, -\eta_5\gamma)}},$$
$$\langle \nabla_3 \rangle, \langle \nabla_3 + \nabla_4 + \alpha\nabla_6 \rangle, \langle \nabla_3 + \alpha\nabla_4 + \nabla_7 \rangle^{O(\alpha)=O(-\alpha)},$$
$$\langle \nabla_3 + \nabla_5 + \alpha\nabla_6 + \beta\nabla_7 \rangle^{O(\alpha,\beta)=O(\alpha,-\beta)}, \langle \nabla_3 + \nabla_6 \rangle,$$
$$\langle \nabla_4 + \alpha\nabla_5 + \beta\nabla_6 + \nabla_8 \rangle^{\substack{O(\alpha,\beta) = O(-i\alpha,-\beta) = \\ O(i\alpha,-\beta) = O(-\alpha,\beta)}},$$
$$\langle \nabla_4 + \nabla_7 \rangle, \langle \nabla_5 + \nabla_6 + \alpha\nabla_7 \rangle^{O(\alpha,\beta)=O(\alpha,-\beta)},$$

$$\langle \nabla_5+\alpha\nabla_6+\nabla_8\rangle, \ \langle \nabla_5+\alpha\nabla_7\rangle^{O(\alpha)=O(-\alpha)}, \ \langle \nabla_6+\nabla_8\rangle, \ \langle \nabla_7\rangle, \ \langle \nabla_8\rangle,$$

which gives the following new algebras:

$\mathbf{N}_{150}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=\alpha e_5$	$e_2e_2=e_4$
		$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=e_4+\mu e_5$	$e_4e_4=e_5$
$\mathbf{N}_{151}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=e_5$	
		$e_2e_2=e_4$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4+\beta e_5$	
$\mathbf{N}_{152}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_4$
		$e_2e_3=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{153}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_4$
		$e_2e_4=e_5$	$e_3e_3=e_4+\beta e_5$	$e_3e_4=\gamma e_5$	
$\mathbf{N}_{154}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_4$	$e_2e_3=\alpha e_5$
		$e_2e_4=\beta e_5$	$e_3e_3=e_4+\gamma e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{155}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	$e_3e_3=e_4$
$\mathbf{N}_{156}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	
		$e_2e_3=e_5$	$e_3e_3=e_4+\alpha e_5$		
$\mathbf{N}_{157}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	
		$e_2e_3=\alpha e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{158}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	
		$e_2e_4=e_5$	$e_3e_3=e_4+\alpha e_5$	$e_3e_4=\beta e_5$	
$\mathbf{N}_{159}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	$e_3e_3=e_4+e_5$
$\mathbf{N}_{160}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_3=e_5$	
		$e_2e_4=\alpha e_5$	$e_3e_3=e_4+\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{161}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_3=e_5$	
		$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{162}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_4=e_5$	
		$e_3e_3=e_4+e_5$	$e_3e_4=\alpha e_5$		
$\mathbf{N}_{163}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_4=e_5$	
		$e_3e_3=e_4+\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{164}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_4=e_5$	
		$e_3e_3=e_4$	$e_3e_4=\alpha e_5$		
$\mathbf{N}_{165}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_3e_3=e_4+e_5$	$e_4e_4=e_5$
$\mathbf{N}_{166}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{167}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_3e_3=e_4$	$e_4e_4=e_5$

3.8. **1-dimensional central extensions of $\mathbf{N}_{13}^{4*}(\lambda)$.** Here we will collect all information about $\mathbf{N}_{13}^{4*}(\lambda)$:

$\mathbf{N}_{13}^{4*}(\lambda)$	$e_1e_1=e_2 \quad e_1e_2=e_3 \quad e_1e_3=e_4 \quad e_2e_2=\lambda e_4$
	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{13}^{4*}(2))=\langle [\Delta_{22}], 4[\Delta_{23}]+[\Delta_{14}], [\Delta_{24}]\rangle$, $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{13}^{4*}(2))=\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{13}^{4*}(2))\oplus\langle [\Delta_{23}], [\Delta_{33}], [\Delta_{34}], [\Delta_{44}]\rangle$ $\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{13}^{4*}(\lambda)_{\lambda\neq 2})=\langle [\Delta_{22}], (3\lambda-2)[\Delta_{23}]+[\Delta_{14}]\rangle$, $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{13}^{4*}(\lambda)_{\lambda\neq 2})=\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{13}^{4*}(\lambda)\oplus\langle [\Delta_{23}], [\Delta_{24}], [\Delta_{33}], [\Delta_{34}], [\Delta_{44}]\rangle$
	$\phi=\begin{pmatrix} x & 0 & 0 & 0 \\ y & x^2 & 0 & 0 \\ z & 2xy & x^3 & 0 \\ t & \lambda y^2+2xz & (\lambda+2)x^2y & x^4 \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll}\nabla_1=[\Delta_{14}]+(3\lambda-2)[\Delta_{23}], & \nabla_2=[\Delta_{22}], & \nabla_3=[\Delta_{23}], & \nabla_4=[\Delta_{24}],\\ \nabla_5=[\Delta_{33}], & \nabla_6=[\Delta_{34}], & \nabla_7=[\Delta_{44}]. &\end{array}$$

Take $\theta=\sum\limits_{i=1}^{7}\alpha_i\nabla_i\in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{13}^{4*}(\lambda))$. Since

$$\phi^T\begin{pmatrix}0 & 0 & 0 & \alpha_1\\ 0 & \alpha_2 & (3\lambda-2)\alpha_1+\alpha_3 & \alpha_4\\ 0 & (3\lambda-2)\alpha_1+\alpha_3 & \alpha_5 & \alpha_6\\ \alpha_1 & \alpha_4 & \alpha_6 & \alpha_7\end{pmatrix}\phi=$$
$$\begin{pmatrix}\alpha^{**} & \alpha^{***} & \alpha^{*} & \alpha_1^*\\ \alpha^{***} & \alpha_2^*+\lambda\alpha^* & (3\lambda-2)\alpha_1^*+\alpha_3^* & \alpha_4^*\\ \alpha^* & (3\lambda-2)\alpha_1^*+\alpha_3^* & \alpha_5^* & \alpha_6^*\\ \alpha_1^* & \alpha_4^* & \alpha_6^* & \alpha_7^*\end{pmatrix},$$

we have

$$\begin{array}{lll}\alpha_1^* &=& (\alpha_1x+\alpha_4y+\alpha_6z+\alpha_7t)x^4,\\ \alpha_2^* &=& \alpha_2x^4+4\lambda(\alpha_6y+\alpha_7z)xy^2+\lambda^2\alpha_7y^4+4(\alpha_4z+(\alpha_3+(3\lambda-2)\alpha_1)y)x^3\\ && +2(4\alpha_6yz+2\alpha_7z^2+(2\alpha_5+\lambda\alpha_4)y^2)x^2\\ && -\lambda((\lambda+2)(\alpha_4y+\alpha_6z+\alpha_7t)y+((\alpha_3+4\lambda\alpha_1)y+\alpha_5z+\alpha_6t)x)x^2,\\ \alpha_3^* &=& [(\lambda+2)(\alpha_4x^2+2\alpha_6xy+2\alpha_7xz+\lambda\alpha_7y^2)y\\ && +((\alpha_3+(3\lambda-2)\alpha_1)x^2+2\alpha_5xy+2\alpha_6xz+\lambda\alpha_6y^2)x]x^2\\ && -(3\lambda-2)(\alpha_1x+\alpha_4y+\alpha_6z+\alpha_7t)x^4,\\ \alpha_4^* &=& (\alpha_4x^2+2\alpha_6xy+2\alpha_7xz+\lambda\alpha_7y^2)x^4,\\ \alpha_5^* &=& (\alpha_5x^2+2(\lambda+2)\alpha_6xy+(\lambda+2)^2\alpha_7y^2)x^4,\\ \alpha_6^* &=& (\alpha_6x+(\lambda+2)\alpha_7y)x^6,\\ \alpha_7^* &=& \alpha_7x^8.\end{array}$$

We are interested in

$$(\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7)\neq(0,0,0,0,0) \text{ and } (\alpha_1,\alpha_4,\alpha_6,\alpha_7)\neq(0,0,0,0).$$

Let us consider the following cases:

1. $\alpha_7=0,\alpha_6=0,\alpha_5=0,\alpha_4=0$, then $\alpha_1\neq 0$, $\alpha_3\neq 0$ and
   (a) if $\lambda\notin\{1,2,4\}$, $(\lambda-4)\alpha_3\neq 4(1-\lambda)(\lambda-2)\alpha_1$, then by choosing $y=\frac{\alpha_2x}{(\lambda-4)\alpha_3+4(\lambda-1)(\lambda-2)\alpha_1}$, we have the family of representatives
   $$\langle\alpha\nabla_1+\nabla_3\rangle_{\alpha\notin\left\{0,\frac{(\lambda-4)}{4(1-\lambda)(\lambda-2)}\right\};\lambda\neq1,2,4};$$
   (b) if $\lambda\notin\{1,2,4\}$, $(\lambda-4)\alpha_3=4(1-\lambda)(\lambda-2)\alpha_1$, $\alpha_2=0$, then we have the family of representatives $\left\langle\frac{\lambda-4}{4(1-\lambda)(\lambda-2)}\nabla_1+\nabla_3\right\rangle_{\lambda\neq1,2,4}$, which we will be jointed with the family from the case (1a);
   (c) if $\lambda\notin\{1,2,4\}$, $(\lambda-4)\alpha_3=4(1-\lambda)(\lambda-2)\alpha_1$, $\alpha_2\neq0$, then by choosing $x=\frac{\alpha_2}{\alpha_3},y=0,z=0,t=0$, we have the family of representatives $\langle(\lambda-4)\nabla_1+4(1-\lambda)(\lambda-2)(\nabla_2+\nabla_3)\rangle_{\lambda\neq1,2,4}$;
   (d) if $\lambda\in\{1,2,4\}$, then by choosing some suitable x and y we have the family of representatives $\langle\alpha\nabla_1+\nabla_3\rangle_{\alpha\neq0,\lambda\in\{1,2,4\}}$, which will be jointed with the family from the case (1a).
2. $\alpha_7=0,\alpha_6=0,\alpha_5=0,\alpha_4\neq0$, then we have
   (a) if $\alpha_3=2(2-\lambda)\alpha_1$, then by choosing $x=4\alpha_4^2, y=-4\alpha_1\alpha_4, z=\alpha_1\alpha_3(4-\lambda)-\alpha_2\alpha_4-\alpha_1^2(8-12\lambda+3\lambda^2), t=0$, we have the representative $\langle\nabla_4\rangle$;

(b) if $\alpha_3 \neq 2(2-\lambda)\alpha_1$, then by choosing

$$x = \frac{\alpha_3+2(\lambda-2)\alpha_1}{\alpha_4},\ y = -\frac{\alpha_1(\alpha_3+2(\lambda-2)\alpha_1)}{\alpha_4^2},$$
$$z = \frac{(2(2-\lambda)\alpha_1-\alpha_3)(\alpha_2\alpha_4+(\lambda-4)\alpha_1\alpha_3+(3\lambda^2-12\lambda+8)\alpha_1^2)}{4\alpha_4^3},\ t=0,$$

we have the representative $\langle\nabla_3+\nabla_4\rangle$.

3. $\alpha_7=0, \alpha_6=0, \alpha_5\neq 0$, then
   (a) if $\alpha_4=0$, then $\alpha_1\neq 0$ and
      (i) if $\lambda\neq 0$, then by choosing
      $$x=\frac{\alpha_1}{\alpha_5}, y=-\frac{\alpha_1\alpha_3}{2\alpha_5^2}, z=\frac{\alpha_1(2\alpha_2\alpha_5+(\lambda-2)\alpha_3^2+4(\lambda^2-3\lambda+2)\alpha_1\alpha_3)}{2\lambda\alpha_5^3}, t=0,$$
      we have the family of representatives $\langle\nabla_1+\nabla_5\rangle_{\lambda\neq 0}$;
      (ii) if $\lambda=0$, then by choosing $x=\frac{\alpha_1}{\alpha_5}, y=-\frac{\alpha_1\alpha_3}{2\alpha_5^2}, z=0, t=0$, we have the family representative $\langle\nabla_1+\alpha\nabla_2+\nabla_5\rangle_{\alpha\neq 0,\lambda=0}$ and the representative $\langle\nabla_1+\nabla_5\rangle_{\lambda=0}$, which will be jointed with the family from the case (3(a)i).
   (b) if $\alpha_4\neq 0$ and $\lambda=0$, then we have the followings:
      (i) if $\alpha_3\alpha_4=2\alpha_1(2\alpha_4+\alpha_5)$, then by choosing
      $x=4\alpha_4^3$, $y=-4\alpha_1\alpha_4^2$, $z=4\alpha_1\alpha_3\alpha_4-\alpha_2\alpha_4^2-4\alpha_1^2(2\alpha_4+\alpha_5)$, $t=0$,
      we have the family of representatives $\langle\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq 0,\lambda=0}$;
      (ii) if $\alpha_3\alpha_4\neq 2\alpha_1(2\alpha_4+\alpha_5)$, then by choosing
      $$x=\frac{\alpha_3\alpha_4-2\alpha_1(2\alpha_4+\alpha_5)}{\alpha_4\alpha_5)}, y=\frac{\alpha_1(2\alpha_1(2\alpha_4+\alpha_5)-\alpha_3\alpha_4)}{\alpha_4^2\alpha_5)},$$
      $$z=\frac{(2\alpha_1(2\alpha_4+\alpha_5)-\alpha_3\alpha_4)(\alpha_2\alpha_4^2-4\alpha_1\alpha_3\alpha_4+4\alpha_1^2(2\alpha_4+\alpha_5))}{4\alpha_4^4\alpha_5}, t=0,$$
      we have the family of representatives $\langle\nabla_3+\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq 0,\lambda=0}$;
   (c) if $\alpha_4\neq 0$ and $\lambda\neq 0$, then we have the followings:
      (i) if $4\alpha_4=\lambda\alpha_5$, $4\lambda(\lambda-4)\alpha_1\alpha_3+\lambda^2\alpha_2\alpha_5+4(3\lambda^3-12\lambda^2+8\lambda+16)\alpha_1^2=0$, $\lambda\alpha_3+2(\lambda^2-2\lambda-4)\alpha_1=0$, then by choosing $x=1, y=-\frac{\alpha_1}{\alpha_4}, z=0, t=0$, we have the family of representatives $\left\langle\frac{\lambda}{4}\nabla_4+\nabla_5\right\rangle_{\lambda\neq 0}$;
      (ii) if $4\alpha_4=\lambda\alpha_5$, $4\lambda(\lambda-4)\alpha_1\alpha_3+\lambda^2\alpha_2\alpha_5+4(3\lambda^3-12\lambda^2+8\lambda+16)\alpha_1^2=0$, $\lambda\alpha_3+2(\lambda^2-2\lambda-4)\alpha_1\neq 0$, then by choosing
      $$x=\frac{\lambda\alpha_3+2(\lambda^2-2\lambda-4)\alpha_1}{\lambda\alpha_5}, y=-\frac{4\alpha_1(\lambda\alpha_3+2(\lambda^2-2\lambda-4)\alpha_1)}{\lambda^2\alpha_5^2}, z=0, t=0,$$
      we have the family of representatives $\left\langle\nabla_3+\frac{\lambda}{4}\nabla_4+\nabla_5\right\rangle_{\lambda\neq 0}$;
      (iii) if $4\alpha_4=\lambda\alpha_5$, $4\lambda(\lambda-4)\alpha_1\alpha_3+\lambda^2\alpha_2\alpha_5+4(3\lambda^3-12\lambda^2+8\lambda+16)\alpha_1^2\neq 0$, then by choosing
      $$x=\frac{\sqrt{4\lambda(\lambda-4)\alpha_1\alpha_3+\lambda^2\alpha_2\alpha_5+4(3\lambda^3-12\lambda^2+8\lambda+16)\alpha_1^2}}{\lambda\alpha_5},$$
      $$y=-\frac{4\alpha_1\sqrt{4\lambda(\lambda-4)\alpha_1\alpha_3+\lambda^2\alpha_2\alpha_5+4(3\lambda^3-12\lambda^2+8\lambda+16)\alpha_1^2}}{\lambda^2\alpha_5^2}, z=0, t=0,$$
      we have the family of representatives $\left\langle\nabla_2+\alpha\nabla_3+\frac{\lambda}{4}\nabla_4+\nabla_5\right\rangle_{\lambda\neq 0}$;
      (iv) if $\lambda\neq 0, 4\alpha_4\neq\lambda\alpha_5$, then by choosing
      $$y=-\frac{\alpha_1}{\alpha_4}x, z=-\frac{\alpha_2\alpha_4^2+(\lambda-4)\alpha_1\alpha_3\alpha_4+\alpha_1^2(4\alpha_5+(3\lambda^2-12\lambda+8)\alpha_4)}{\alpha_4^2(4\alpha_4-\lambda\alpha_5)}x, t=0,$$
      we have two families of representatives
      $$\langle\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq\frac{\lambda}{4}} \text{ and } \langle\nabla_3+\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq\frac{\lambda}{4}}$$
      depending on $\alpha_3\alpha_4-2\alpha_1\alpha_5+2(\lambda-2)\alpha_1\alpha_4=0$ or not. These families will be jointed with representatives from cases (3(c)i) and (3(c)ii).
4. $\alpha_7=0, \alpha_6\neq 0$, then by choosing $y=-\frac{\alpha_4}{2\alpha_6}x, z=-\frac{\alpha_4^2-2\alpha_1\alpha_6}{2\alpha_6^2}x$, we have $\alpha_1^*=0, \alpha_4^*=0$. Since we can suppose that $\alpha_1=0, \alpha_4=0$ and

(a) if $\lambda \neq 0, \alpha_3 = 0$, then by choosing $t = \frac{\alpha_2}{\lambda\alpha_6}x$, we have the representatives $\langle\nabla_6\rangle_{\lambda\neq 0}$ and $\langle\nabla_5+\nabla_6\rangle_{\lambda\neq 0}$ depending on $\alpha_5 = 0$ or not;

(b) if $\lambda \neq 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_3}{\alpha_6}}, t = \frac{\alpha_2\sqrt{\alpha_3}}{\lambda\alpha_6\sqrt{\alpha_6}}$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_5+\nabla_6\rangle_{\lambda\neq 0}$;

(c) if $\lambda = 0, \alpha_2 = 0, \alpha_3 = 0$, then we have the representatives $\langle\nabla_6\rangle_{\lambda=0}$ and $\langle\nabla_5+\nabla_6\rangle_{\lambda=0}$ depending on $\alpha_5 = 0$ or not, which will be jointed with representatives from the case (4a);

(d) if $\lambda = 0, \alpha_2 = 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_3}{\alpha_6}}, t = 0$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_5+\nabla_6\rangle_{(\lambda=0)}$, which will be jointed with the family of representatives from the case (4b);

(e) if $\lambda = 0, \alpha_2 \neq 0$, then by choosing $x = \sqrt[3]{\frac{\alpha_2}{\alpha_6}}, t = 0$, we have the family of representatives $\langle\nabla_2+\alpha\nabla_3+\beta\nabla_5+\nabla_6\rangle_{\lambda=0}$.

5. $\alpha_7 \neq 0, \lambda \neq -2$, then by choosing

$$y = -\frac{\alpha_6}{\alpha_7(\lambda+2)}x,\ z = \frac{2(\lambda+2)^2\alpha_4\alpha_7-(\lambda+4)\alpha_6^2}{2(\lambda+2)^2\alpha_7^2}x,$$
$$t = \frac{(\lambda^2+6\lambda+8)\alpha_4\alpha_6\alpha_7-2(\lambda+2)^2\alpha_1\alpha_7^2-(\lambda+4)\alpha_6^3}{2(\lambda+2)^2\alpha_7^3}x,$$

we have $\alpha_1^* = 0, \alpha_4^* = 0, \alpha_6^* = 0$. Now we can suppose that $\alpha_1 = 0, \alpha_4 = 0, \alpha_6 = 0$ then we have

(a) if $\alpha_3 = 0, \alpha_5 = 0, \alpha_2 = 0$, then we have the representative $\langle\nabla_7\rangle_{\lambda\neq -2}$;

(b) if $\alpha_3 = 0, \alpha_5 = 0, \alpha_2 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_2\alpha_7^{-1}}$, we have the representative $\langle\nabla_2+\nabla_7\rangle_{\lambda\neq -2}$;

(c) if $\alpha_5 \neq 0$, then by choosing $x = \sqrt{\alpha_5\alpha_7^{-1}}$, $y = -\frac{\alpha_3}{2\sqrt{\alpha_5\alpha_7}}$, $z = \frac{\alpha_3^2}{4\sqrt{\alpha_5^3\alpha_7}}$, $t = 0$, we have the family of representatives $\langle\alpha\nabla_2+\nabla_5+\nabla_7\rangle_{\lambda\neq -2}$.

(d) if $\alpha_3 \neq 0, \alpha_5 = 0$ then by choosing $x = \sqrt[3]{\alpha_3\alpha_7^{-1}}$, we have the family of representatives $\langle\alpha\nabla_2+\nabla_3+\nabla_7\rangle_{\lambda\neq -2}$.

6. $\alpha_7 \neq 0, \lambda = -2$, then

(a) if $\alpha_6 = 0, \alpha_5 = 0$, then by choosing $z = \frac{y^2}{x} - \frac{\alpha_4 x}{2\alpha_7}, t = -\frac{x\alpha_1+y\alpha_4}{\alpha_7}$, we have $\alpha_1^* = 0$ and $\alpha_4^* = 0$. Now consider the followings:

(i) if $\alpha_3 = 8\alpha_1, \alpha_2\alpha_7-\alpha_4^2 = 0$, then we have the representative $\langle\nabla_7\rangle_{\lambda=-2}$, which will be jointed with the representative from the case (5a);

(ii) if $\alpha_3 = 8\alpha_1, \alpha_2\alpha_7-\alpha_4^2 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_2\alpha_7-\alpha_4^2}{\alpha_7^2}}, y = 0$, we have the representative $\langle\nabla_2+\nabla_7\rangle_{\lambda=-2}$, which will be jointed with the representative from the case (5b);

(iii) if $\alpha_3 \neq 8\alpha_1$, then by choosing $x = \sqrt[3]{\frac{\alpha_3-8\alpha_1}{\alpha_7}}, y = \frac{\alpha_2\alpha_7-\alpha_4^2}{48\alpha_1\alpha_7-6\alpha_3\alpha_7}x$, we have the representative $\langle\nabla_3+\nabla_7\rangle_{\lambda=-2}$.

(b) if $\alpha_6 = 0, \alpha_5 \neq 0$, then by choosing

$$x = \sqrt{\frac{\alpha_5}{\alpha_7}}, y = \frac{8\alpha_1-\alpha_3}{2\sqrt{\alpha_5\alpha_7}}, z = \frac{\alpha_7(\alpha_3-8\alpha_1)^2-2\alpha_4\alpha_5^2}{4\alpha_5\alpha_7\sqrt{\alpha_5\alpha_7}}, t = \frac{\alpha_3\alpha_4-2\alpha_1(4\alpha_4+\alpha_5)}{2\alpha_7\sqrt{\alpha_5\alpha_7}},$$

we have the family of representatives $\langle\alpha\nabla_2+\nabla_5+\nabla_7\rangle_{\lambda=-2}$, which will be jointed with the representative from the case (5c).

(c) if $\alpha_6 \neq 0$, then we have the following cases:

(i) if $\alpha_5\alpha_7 = \alpha_6^2, 8\alpha_1\alpha_7+\alpha_4\alpha_6 = \alpha_3\alpha_7$, then by choosing

$$x = \frac{\alpha_6}{\alpha_7}, y = 0, z = -\frac{\alpha_4\alpha_6}{2\alpha_7^2}, t = \frac{\alpha_6(\alpha_4\alpha_6-2\alpha_1\alpha_7)}{2\alpha_7^3},$$

we have the family of representatives $\langle \alpha\nabla_2+\nabla_5+\nabla_6+\nabla_7\rangle_{\lambda=-2}$;

(ii) if $\alpha_5\alpha_7=\alpha_6^2, 8\alpha_1\alpha_7+\alpha_4\alpha_6\neq\alpha_3\alpha_7$, then by choosing

$$x=\frac{\alpha_6}{\alpha_7}, y=\frac{\alpha_6(\alpha_2\alpha_7-\alpha_4^2-2\alpha_1\alpha_6)}{6\alpha_7(\alpha_4\alpha_6+8\alpha_1\alpha_7-\alpha_3\alpha_7))},$$
$$z=\frac{y^2}{x}-\frac{\alpha_4 x}{2\alpha_7}-\frac{\alpha_6 y}{\alpha_7}, t=-\frac{x\alpha_1+y\alpha_4+z\alpha_6}{\alpha_7},$$

we have the family of representatives

$$\langle \alpha\nabla_3+\nabla_5+\nabla_6+\nabla_7\rangle_{\alpha\neq0,\lambda=-2};$$

(iii) if $\alpha_5\alpha_7-\alpha_6^2\neq 0$, then by choosing

$$x=\frac{\alpha_6}{\alpha_7}, y=\frac{\alpha_6(\alpha_4\alpha_6+8\alpha_1\alpha_7-\alpha_3\alpha_7)}{2\alpha_7(-\alpha_6^2+\alpha_5\alpha_7)}, z=\frac{y^2}{x}-\frac{\alpha_4}{2\alpha_7}x-\frac{\alpha_6}{\alpha_7}y,$$
$$t=\frac{(\alpha_4\alpha_6-2\alpha_1\alpha_7)x^2-2\alpha_6\alpha_7y^2+2(\alpha_6^2-\alpha_4\alpha_7)xy}{2\alpha_7^2x},$$

we have the family of representatives

$$\langle \alpha\nabla_2+\beta\nabla_5+\nabla_6+\nabla_7\rangle_{\beta\neq1,\lambda=-2},$$

which will be jointed with the family from the case (6(c)i).

Summarizing all cases we have the following distinct orbits:

$$\langle(\lambda-4)\nabla_1+4(1-\lambda)(\lambda-2)(\nabla_2+\nabla_3)\rangle_{\lambda\notin\{1;2;4\}}, \langle\nabla_1+\alpha\nabla_2+\nabla_5\rangle_{\lambda=0,\alpha\neq0},$$
$$\langle\alpha\nabla_1+\nabla_3\rangle_{\alpha\neq0}, \langle\nabla_1+\nabla_5\rangle, \left\langle\nabla_2+\alpha\nabla_3+\tfrac{\lambda}{4}\nabla_4+\nabla_5\right\rangle_{\lambda\neq0}^{O(\alpha)=O(-\alpha)},$$
$$\langle\nabla_2+\alpha\nabla_3+\beta\nabla_5+\nabla_6\rangle_{\lambda=0}^{O(\alpha,\beta)=O(\eta_3^2\alpha,-\eta\beta)=O(-\eta_3\alpha,\eta_3^2\beta)},$$
$$\langle\alpha\nabla_2+\nabla_3+\nabla_7\rangle_{\lambda\neq-2}^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)}, \langle\alpha\nabla_2+\beta\nabla_5+\nabla_6+\nabla_7\rangle_{\lambda=-2},$$
$$\langle\alpha\nabla_2+\nabla_5+\nabla_7\rangle, \langle\nabla_2+\nabla_7\rangle, \langle\nabla_3+\nabla_4\rangle, \langle\nabla_3+\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq0},$$
$$\langle\nabla_3+\alpha\nabla_5+\nabla_6\rangle, \langle\alpha\nabla_3+\nabla_5+\nabla_6+\nabla_7\rangle_{\alpha\neq0,\lambda=-2}, \langle\nabla_3+\nabla_7\rangle_{\lambda=-2}, \langle\nabla_4\rangle_{\lambda\neq2},$$
$$\langle\alpha\nabla_4+\nabla_5\rangle_{\alpha\neq0}, \langle\nabla_5+\nabla_6\rangle, \langle\nabla_6\rangle, \langle\nabla_7\rangle.$$

Now we have the following new algebras

$\mathbf{N}_{168}^{\lambda\neq1;2;4}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=(\lambda-4)e_5$
		$e_2e_2=\lambda e_4+4(1-\lambda)(\lambda-2)e_5$		$e_2e_3=-\lambda(\lambda+2)e_5$	
$\mathbf{N}_{169}^{\alpha\neq0}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$
		$e_2e_2=\alpha e_5$	$e_2e_3=-2e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{170}^{\lambda,\alpha\neq0}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_1e_4=\alpha e_5$	$e_2e_2=\lambda e_4$	$e_2e_3=(1+\alpha(3\lambda-2))e_5$	
$\mathbf{N}_{171}^{\lambda}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$
		$e_2e_2=\lambda e_4$	$e_2e_3=(3\lambda-2)e_5$		$e_3e_3=e_5$
$\mathbf{N}_{172}^{\lambda\neq0,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=\lambda e_4+e_5$
		$e_2e_3=\alpha e_5$	$e_2e_4=\frac{\lambda}{4}e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{173}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=e_5$
		$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{174}^{\lambda\neq-2,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4+\alpha e_5$	$e_2e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{175}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=-2e_4+\alpha e_5$
		$e_3e_3=\beta e_5$	$e_3e_4=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{176}^{\lambda,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4+\alpha e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{177}^{\lambda}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4+e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{178}^{\lambda}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=\lambda e_4$
		$e_2e_3=e_5$	$e_2e_4=e_5$		
$\mathbf{N}_{179}^{\lambda,\alpha\neq0}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=\lambda e_4$

		$e_2e_3 = e_5$	$e_2e_4 = \alpha e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{180}^{\lambda,\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \lambda e_4$
		$e_2e_3 = e_5$	$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{181}^{\alpha\neq 0}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = -2e_4$
		$e_2e_3 = \alpha e_5$	$e_3e_3 = e_5$	$e_3e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{182}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = -2e_4$	$e_2e_3 = e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{183}^{\lambda\neq 2}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_2e_4 = e_5$		
$\mathbf{N}_{184}^{\lambda,\alpha\neq 0}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_2e_4 = \alpha e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{185}^{\lambda}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_3e_3 = e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{186}^{\lambda}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{187}^{\lambda}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_4e_4 = e_5$		

3.9. **1-dimensional central extensions of $\mathbf{N}_{14}^{4*}$.** Here we will collect all information about $\mathbf{N}_{14}^{4*}$:

$\mathbf{N}_{14}^{4*}$	$e_1e_2 = e_3$ $e_1e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{14}^{4*}) = \langle [\Delta_{11}], [\Delta_{22}], [\Delta_{23}], [\Delta_{33}] \rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{14}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{14}^{4*}) \oplus \langle [\Delta_{14}], [\Delta_{24}], [\Delta_{34}], [\Delta_{44}] \rangle$	$\phi = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & r & xq & 0 \\ t & s & xr & x^2q \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{11}], & \nabla_2 = [\Delta_{14}], & \nabla_3 = [\Delta_{22}], & \nabla_4 = [\Delta_{23}], \\ \nabla_5 = [\Delta_{24}], & \nabla_6 = [\Delta_{33}], & \nabla_7 = [\Delta_{34}], & \nabla_8 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{14}^{4*})$. Since

$$\phi^T \begin{pmatrix} \alpha_1 & 0 & 0 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & \alpha_4 & \alpha_6 & \alpha_7 \\ \alpha_2 & \alpha_5 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha_1^* & \alpha^* & \alpha^{**} & \alpha_2^* \\ \alpha^* & \alpha_3^* & \alpha_4^* & \alpha_5^* \\ \alpha^{**} & \alpha_4^* & \alpha_6^* & \alpha_7^* \\ \alpha_2^* & \alpha_5^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

we have

$$\begin{array}{rcl} \alpha_1^* &=& \alpha_1 x^2 + 2\alpha_2 xt + \alpha_8 t^2, \\ \alpha_2^* &=& (\alpha_2 x + \alpha_8 t) x^2 q, \\ \alpha_3^* &=& (\alpha_3 q + \alpha_4 r + \alpha_5 s) q + (\alpha_4 q + \alpha_6 r + \alpha_7 s) r + (\alpha_5 q + \alpha_7 r + \alpha_8 s) s, \\ \alpha_4^* &=& (\alpha_4 q + \alpha_6 r + \alpha_7 s) xq + (\alpha_5 q + \alpha_7 r + \alpha_8 s) xr, \\ \alpha_5^* &=& (\alpha_5 q + \alpha_7 r + \alpha_8 s) x^2 q, \\ \alpha_6^* &=& (\alpha_6 q^2 + 2\alpha_7 qr + \alpha_8 r^2) x^2, \\ \alpha_7^* &=& (\alpha_7 q + \alpha_8 r) x^3 q, \\ \alpha_8^* &=& \alpha_8 x^4 q^2. \end{array}$$

We are interested in $(\alpha_2, \alpha_5, \alpha_7, \alpha_8) \neq (0,0,0,0)$. Let us consider the following cases:

1. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 = 0$, then $\alpha_2 \neq 0$ and we have

(a) if $\alpha_6 = 0, \alpha_4 = 0, \alpha_3 = 0$, then by choosing $x = 2\alpha_2, q = 1, r = 0, s = 0, t = -\alpha_1$, we have the representative $\langle \nabla_2 \rangle$;

(b) if $\alpha_6 = 0, \alpha_4 = 0, \alpha_3 \neq 0$, then by choosing $x = \alpha_3, q = \alpha_2\alpha_3^2, r = 0, s = 0, t = -\frac{\alpha_1\alpha_3}{2\alpha_2}$, we have the representative $\langle \nabla_2 + \nabla_3 \rangle$;

(c) if $\alpha_6 = 0, \alpha_4 \neq 0$, then by choosing $x = \alpha_4, q = \alpha_2\alpha_4, r = -\frac{\alpha_2\alpha_3}{2}, s = 0, t = -\frac{\alpha_1\alpha_4}{2\alpha_2}$, we have the representative $\langle \nabla_2 + \nabla_4 \rangle$;

(d) if $\alpha_6 \neq 0, \alpha_3\alpha_6 - \alpha_4^2 = 0$, then by choosing $x = \alpha_6, q = \alpha_2, r = -\frac{\alpha_2\alpha_4}{\alpha_6}, s = 0, t = -\frac{\alpha_1\alpha_6}{2\alpha_2}$, we have the representative $\langle \nabla_2 + \nabla_6 \rangle$;

(e) if $\alpha_6 \neq 0, \alpha_3\alpha_6 - \alpha_4^2 \neq 0$, then by choosing

$$x = \frac{\sqrt{\alpha_3\alpha_6 - \alpha_4^2}}{\alpha_6},\ q = \frac{\alpha_2\sqrt{\alpha_3\alpha_6 - \alpha_4^2}}{\alpha_6^2},\ r = -\frac{\alpha_2\alpha_4\sqrt{\alpha_3\alpha_6 - \alpha_4^2}}{\alpha_6^3},$$

$$s = 0, t = -\frac{\alpha_1\sqrt{\alpha_3\alpha_6 - \alpha_4^2}}{2\alpha_2\alpha_6},$$

we have the representative $\langle \nabla_2 + \nabla_3 + \nabla_6 \rangle$.

2. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 \neq 0$, then we have

(a) if $\alpha_6 = 0, \alpha_2 = 0$, then by choosing

$$x = 1, r = -\frac{\alpha_4}{\alpha_5}q, s = \frac{2\alpha_4^2 - \alpha_3\alpha_5}{2\alpha_5^2}q, t = 0,$$

we have the representatives $\langle \nabla_5 \rangle$ and $\langle \nabla_1 + \nabla_5 \rangle$ depending on whether $\alpha_1 = 0$ or not;

(b) if $\alpha_6 = 0, \alpha_2 \neq 0$, then by choosing

$$x = \alpha_5, q = \alpha_2, r = -\frac{\alpha_2\alpha_4}{\alpha_5}, s = \frac{\alpha_2(2\alpha_4^2 - \alpha_3\alpha_5)}{2\alpha_5^2}, t = -\frac{\alpha_1\alpha_5}{2\alpha_2},$$

we have the representatives $\langle \nabla_2 + \nabla_5 \rangle$;

(c) if $\alpha_6 \neq 0, \alpha_5 = -\alpha_6$, then we have the following subcases:

(i) if $\alpha_2 = 0, \alpha_4 = 0, \alpha_1 = 0$, then we have the representative $\langle \nabla_5 - \nabla_6 \rangle$;

(ii) if $\alpha_2 = 0, \alpha_4 = 0, \alpha_1 \neq 0$, then by choosing

$$x = 1, q = \sqrt{\frac{\alpha_1}{\alpha_5}}, r = 0, s = -\frac{\alpha_3\sqrt{\alpha_1}}{2\alpha_5\sqrt{\alpha_5}}, t = 0,$$

we have the representative $\langle \nabla_1 + \nabla_5 - \nabla_6 \rangle$;

(iii) if $\alpha_2 = 0, \alpha_4 \neq 0, \alpha_1 = 0$, then by choosing

$$x = \frac{\alpha_4}{\alpha_5}, q = 1, r = 0, s = -\frac{\alpha_3}{2\alpha_5}, t = 0,$$

we have the representative $\langle \nabla_4 + \nabla_5 - \nabla_6 \rangle$;

(iv) if $\alpha_2 = 0, \alpha_4 \neq 0, \alpha_1 \neq 0$, then by choosing

$$x = \frac{\alpha_4}{\alpha_5}, q = \sqrt{\frac{\alpha_1}{\alpha_5}}, r = 0, s = -\frac{\alpha_3\sqrt{\alpha_1}}{2\alpha_5\sqrt{\alpha_5}}, t = 0,$$

we have the representative $\langle \nabla_1 + \nabla_4 + \nabla_5 - \nabla_6 \rangle$;

(v) if $\alpha_2 \neq 0, \alpha_4 = 0$, then by choosing

$$x = \alpha_5, q = \alpha_2, r = 0, s = -\frac{\alpha_2\alpha_3}{2\alpha_5}, t = -\frac{\alpha_1\alpha_5}{2\alpha_2},$$

we have the representative $\langle \nabla_2 + \nabla_5 - \nabla_6 \rangle$;

(vi) if $\alpha_2 \neq 0, \alpha_4 \neq 0$, then by choosing

$$x = \frac{\alpha_4}{\alpha_5}, q = \frac{\alpha_2\alpha_4}{\alpha_5^2}, r = 0, s = -\frac{\alpha_2\alpha_3\alpha_4}{2\alpha_5^3}, t = -\frac{\alpha_1\alpha_4}{2\alpha_2\alpha_5},$$

we have the representative $\langle \nabla_2 + \nabla_4 + \nabla_5 - \nabla_6 \rangle$.

(d) if $\alpha_6 \neq 0, \alpha_5 \neq -\alpha_6$, then we have the following subcases:

(i) if $\alpha_2 = 0, \alpha_1 = 0$, then by choosing

$$x = 1, q = 1, s = \frac{\alpha_4^2(2\alpha_5 + \alpha_6) - \alpha_3(\alpha_5 + \alpha_6)^2}{2\alpha_5(\alpha_5 + \alpha_6)^2}, r = -\frac{\alpha_4}{\alpha_5 + \alpha_6}, t = 0,$$

we have the family of representatives $\langle \nabla_5 + \alpha\nabla_6 \rangle_{\alpha \neq 0, -1}$, which will be jointed with the cases (2a) and (2(c)i);

(ii) if $\alpha_2 = 0, \alpha_1 \neq 0$, then by choosing

$$x=1, q=\sqrt{\frac{\alpha_1}{\alpha_5}},\ r=-\frac{\alpha_4\sqrt{\alpha_1}}{(\alpha_5+\alpha_6)\sqrt{\alpha_5}},$$
$$s=\frac{(\alpha_4^2(2\alpha_5+\alpha_6)-\alpha_3(\alpha_5+\alpha_6)^2)\sqrt{\alpha_1}}{2\alpha_5(\alpha_5+\alpha_6)^2\sqrt{\alpha_5}}, t=0,$$

we have the family of representatives $\langle\nabla_1+\nabla_5+\alpha\nabla_6\rangle_{\alpha\neq0,-1}$, which will be jointed with the cases (2a) and (2(c)ii);

(iii) if $\alpha_2\neq0$, then by choosing
$$x=\alpha_5, q=\alpha_2,\ r=-\frac{\alpha_2\alpha_4}{\alpha_5+\alpha_6},$$
$$s=\frac{\alpha_2(\alpha_4^2(2\alpha_5+\alpha_6)-\alpha_3(\alpha_5+\alpha_6)^2)}{2\alpha_5(\alpha_5+\alpha_6)^2}, t=-\frac{\alpha_1\alpha_5}{2\alpha_2},$$
we have the family of representatives $\langle\nabla_2+\nabla_5+\alpha\nabla_6\rangle_{\alpha\neq0,-1}$, which will be jointed with the cases (2b) and (2(c)v).

3. $\alpha_8=0,\alpha_7\neq0$ then by choosing $r=-\frac{\alpha_5}{\alpha_7}q, s=\frac{\alpha_5\alpha_6-\alpha_4\alpha_7}{\alpha_7^2}q$, we have $\alpha_4^*=\alpha_5^*=0$. Therefore, we can suppose that $\alpha_4=0,\alpha_5=0$, thus we have
   (a) if $\alpha_2=0,\alpha_1=0,\alpha_3=0$, then we have the representatives $\langle\nabla_7\rangle$ and $\langle\nabla_6+\nabla_7\rangle$ depending on whether $\alpha_6=0$ or not;
   (b) if $\alpha_2=0,\alpha_1=0,\alpha_3\neq0$, then by choosing $x=\sqrt[3]{\frac{\alpha_3}{\alpha_7}}, q=1, r=0, s=0, t=0$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_6+\nabla_7\rangle$;
   (c) if $\alpha_2=0,\alpha_1\neq0,\alpha_3=0$, then we have the representatives $\langle\nabla_1+\nabla_7\rangle$ and $\langle\nabla_1+\nabla_6+\nabla_7\rangle$ depending on whether $\alpha_6=0$ or not;
   (d) if $\alpha_2=0,\alpha_1\neq0,\alpha_3\neq0$, then by choosing
   $$x=\sqrt[3]{\frac{\alpha_3}{\alpha_7}}, q=\sqrt[6]{\frac{\alpha_1^3}{\alpha_3\alpha_7^2}}, r=0, s=0, t=0,$$
   we have the family of representatives $\langle\nabla_1+\nabla_3+\alpha\nabla_6+\nabla_7\rangle$;
   (e) if $\alpha_2\neq0,\alpha_3=0$, then by choosing $q=\frac{\alpha_2}{\alpha_7}, r=0, s=0, t=-\frac{\alpha_1}{2\alpha_2}x$, we have the representatives $\langle\nabla_2+\nabla_7\rangle$ and $\langle\nabla_2+\nabla_6+\nabla_7\rangle$ depending on whether $\alpha_6=0$ or not;
   (f) if $\alpha_2\neq0,\alpha_3\neq0$, then by choosing $x=\sqrt[3]{\frac{\alpha_3}{\alpha_7}}, q=\frac{\alpha_2}{\alpha_7}, r=0, s=0, t=-\frac{\alpha_1\sqrt[3]{\alpha_3}}{2\alpha_2\sqrt[3]{\alpha_7}}$ we have the family of representatives $\langle\nabla_2+\nabla_3+\alpha\nabla_6+\nabla_7\rangle$.
4. $\alpha_8\neq0$ then by choosing $r=-\frac{\alpha_7}{\alpha_8}q, s=\frac{\alpha_7^2-\alpha_5\alpha_8}{\alpha_8^2}q, t=-\frac{\alpha_2}{\alpha_8}x$, we have $\alpha_2^*=\alpha_5^*=\alpha_7^*=0$. Therefore, we can suppose that $\alpha_2=0,\alpha_5=0,\alpha_7=0$, then we have
   (a) if $\alpha_1=0,\alpha_3=0,\alpha_4=0$, then we have the representatives $\langle\nabla_8\rangle$ and $\langle\nabla_6+\nabla_8\rangle$ depending on whether $\alpha_6=0$ or not;
   (b) if $\alpha_1=0,\alpha_3=0,\alpha_4\neq0$, then by choosing $x=\sqrt[3]{\frac{\alpha_4}{\alpha_8}}, q=1, r=0, s=0, t=0$, we have the family of representatives $\langle\nabla_4+\alpha\nabla_6+\nabla_8\rangle$;
   (c) if $\alpha_1=0,\alpha_3\neq0$, then by choosing $x=\sqrt[4]{\frac{\alpha_3}{\alpha_8}}, q=1, r=0, s=0, t=0$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_4+\beta\nabla_6+\nabla_8\rangle$;
   (d) if $\alpha_1\neq0,\alpha_3=0,\alpha_4=0$, then we have the representatives $\langle\nabla_1+\nabla_8\rangle$ and $\langle\nabla_1+\nabla_6+\nabla_8\rangle$ depending on whether $\alpha_6=0$ or not;
   (e) if $\alpha_1\neq0,\alpha_3=0,\alpha_4\neq0$, then by choosing $x=\sqrt[3]{\frac{\alpha_4}{\alpha_8}}, q=\sqrt[6]{\frac{\alpha_1^3}{\alpha_4^2\alpha_8}}, r=0, s=0, t=0$, we have the family of representative $\langle\nabla_1+\nabla_4+\alpha\nabla_6+\nabla_8\rangle$;
   (f) if $\alpha_1\neq0,\alpha_3\neq0$, then by choosing $x=\sqrt[4]{\frac{\alpha_3}{\alpha_8}}, q=\frac{\sqrt{\alpha_1}}{\sqrt[4]{\alpha_3\alpha_8}}, r=0, s=0, t=0$, we have the family of representative $\langle\nabla_1+\nabla_3+\alpha\nabla_4+\beta\nabla_6+\nabla_8\rangle$.

Summarizing, we have the following distinct orbits:

$$\langle\nabla_1+\nabla_3+\alpha\nabla_4+\beta\nabla_6+\nabla_8\rangle^{O(\alpha,\beta)=O(i\alpha,-\beta)=O(-i\alpha,-\beta)=O(-\alpha,\beta)},$$
$$\langle\nabla_1+\nabla_3+\alpha\nabla_6+\nabla_7\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_1+\nabla_4+\nabla_5-\nabla_6\rangle,$$
$$\langle\nabla_1+\nabla_4+\alpha\nabla_6+\nabla_8\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_1+\nabla_5+\alpha\nabla_6\rangle,\ \langle\nabla_1+\nabla_6+\nabla_7\rangle,$$
$$\langle\nabla_1+\nabla_6+\nabla_8\rangle,\ \langle\nabla_1+\nabla_7\rangle,\ \langle\nabla_1+\nabla_8\rangle,\ \langle\nabla_2\rangle,\ \langle\nabla_2+\nabla_3\rangle,\ \langle\nabla_2+\nabla_3+\nabla_6\rangle,$$
$$\langle\nabla_2+\nabla_3+\alpha\nabla_6+\nabla_7\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_2+\nabla_4\rangle,\ \langle\nabla_2+\nabla_4+\nabla_5-\nabla_6\rangle,$$
$$\langle\nabla_2+\nabla_5+\alpha\nabla_6\rangle,\ \langle\nabla_2+\nabla_6\rangle,\ \langle\nabla_2+\nabla_6+\nabla_7\rangle,\ \langle\nabla_2+\nabla_7\rangle,$$
$$\langle\nabla_3+\alpha\nabla_4+\beta\nabla_6+\nabla_8\rangle^{O(\alpha,\beta)=O(i\alpha,-\beta)=O(-i\alpha,-\beta)=O(-\alpha,\beta)},$$
$$\langle\nabla_3+\alpha\nabla_6+\nabla_7\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_4+\nabla_5-\nabla_6\rangle,$$
$$\langle\nabla_4+\alpha\nabla_6+\nabla_8\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_5+\alpha\nabla_6\rangle,\ \langle\nabla_6+\nabla_7\rangle,\ \langle\nabla_6+\nabla_8\rangle,$$
$$\langle\nabla_7\rangle,\ \langle\nabla_8\rangle,$$

which gives the following new algebras:

$\mathbf{N}_{188}^{\alpha,\beta}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=e_5$	
		$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{189}^{\alpha}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$		
		$e_2e_2=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{190}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$		
		$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_3=-e_5$		
$\mathbf{N}_{191}^{\alpha}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$		
		$e_2e_3=e_5$	$e_3e_3=\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{191}^{\alpha}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$
$\mathbf{N}_{192}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_3e_3=e_5$	$e_3e_4=e_5$
$\mathbf{N}_{193}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{194}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{195}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_4e_4=e_5$	
$\mathbf{N}_{196}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$		
$\mathbf{N}_{197}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_2e_2=e_5$	
$\mathbf{N}_{198}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_2e_2=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{199}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$		
		$e_2e_2=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{200}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_2e_3=e_5$	
$\mathbf{N}_{201}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$		
		$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_3=-e_5$		
$\mathbf{N}_{202}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$
$\mathbf{N}_{203}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{204}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_3e_3=e_5$	$e_3e_4=e_5$
$\mathbf{N}_{205}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{206}^{\alpha,\beta}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=e_5$		
		$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{207}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$
$\mathbf{N}_{208}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_3=-e_5$
$\mathbf{N}_{209}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_3=e_5$	$e_3e_3=\alpha e_5$	$e_4e_4=e_5$
$\mathbf{N}_{210}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{211}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_3e_3=e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{212}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_3e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{213}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{214}$	:	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_4e_4=e_5$		

3.10. **1-dimensional central extensions of $\mathbf{N}_{15}^{4*}$.** Here we will collect all information about $\mathbf{N}_{15}^{4*}$:

$\mathbf{N}_{15}^{4*}$	$e_1e_2 = e_3$ $e_1e_3 = e_4$ $e_2e_2 = e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{15}^{4*}) = \langle [\Delta_{11}], [\Delta_{22}], [\Delta_{23}], [\Delta_{33}] \rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{15}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{15}^{4*}) \oplus \langle [\Delta_{14}], [\Delta_{24}], [\Delta_{34}], [\Delta_{44}] \rangle$	$\phi = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & x^2 & 0 & 0 \\ 0 & r & x^3 & 0 \\ t & s & xr & x^4 \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{11}], & \nabla_2 = [\Delta_{14}], & \nabla_3 = [\Delta_{22}], & \nabla_4 = [\Delta_{23}], \\ \nabla_5 = [\Delta_{24}], & \nabla_6 = [\Delta_{33}], & \nabla_7 = [\Delta_{34}], & \nabla_8 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{15}^{4*})$. Since

$$\phi^T \begin{pmatrix} \alpha_1 & 0 & 0 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & \alpha_4 & \alpha_6 & \alpha_7 \\ \alpha_2 & \alpha_5 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha_1^* & \alpha^* & \alpha^{**} & \alpha_2^* \\ \alpha^* & \alpha_3^* + \alpha^{**} & \alpha_4^* & \alpha_5^* \\ \alpha^{**} & \alpha_4^* & \alpha_6^* & \alpha_7^* \\ \alpha_2^* & \alpha_5^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

we have

$$\begin{array}{rcl} \alpha_1^* & = & \alpha_1 x^2 + 2\alpha_2 xt + \alpha_8 t^2, \\ \alpha_2^* & = & (\alpha_2 x + \alpha_8 t) x^4, \\ \alpha_3^* & = & x^4\alpha_3 + 2rx^2\alpha_4 + 2sx^2\alpha_5 + r^2\alpha_6 + 2rs\alpha_7 + \\ & & \qquad s^2\alpha_8 - x(rx\alpha_2 + tx^2\alpha_7 + rt\alpha_8), \\ \alpha_4^* & = & (\alpha_4 x^2 + \alpha_6 r + \alpha_7 s)x^3 + (\alpha_5 x^2 + \alpha_7 r + \alpha_8 s)xr, \\ \alpha_5^* & = & (\alpha_5 x^2 + \alpha_7 r + \alpha_8 s)x^4, \\ \alpha_6^* & = & (\alpha_6 x^4 + 2\alpha_7 x^2 r + \alpha_8 r^2)x^2, \\ \alpha_7^* & = & (\alpha_7 x^2 + \alpha_8 r)x^5, \\ \alpha_8^* & = & \alpha_8 x^8. \end{array}$$

We are interested in $(\alpha_2, \alpha_5, \alpha_7, \alpha_8) \neq (0,0,0,0)$. Let us consider the following cases:

1. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 = 0$, then $\alpha_2 \neq 0$ and we have
   (a) if $\alpha_6 = 0, \alpha_4 = 0$, then by choosing $x = 2\alpha_2, r = 4\alpha_2\alpha_3, s = 0, t = -\alpha_1$, we have the representative $\langle \nabla_2 \rangle$;
   (b) if $\alpha_6 = 0, \alpha_4 \neq 0, \alpha_2 = 2\alpha_4$, then we have the representatives $\langle 2\nabla_2 + \nabla_4 \rangle$ and $\langle 2\nabla_2 + \nabla_3 + \nabla_4 \rangle$ depending on whether $\alpha_3 = 0$ or not;
   (c) if $\alpha_6 = 0, \alpha_4 \neq 0, \alpha_2 \neq 2\alpha_4$, then by choosing $x = \alpha_2 - 2\alpha_4, r = \alpha_3(\alpha_2 - 2\alpha_4), s = 0, t = \frac{\alpha_1(2\alpha_4 - \alpha_2)}{2\alpha_2}$, we have the family of representatives $\langle \nabla_2 + \alpha\nabla_4 \rangle_{\alpha \neq 0, \frac{1}{2}}$, which will be jointed with representatives from the cases (1a) and (1b);
   (d) if $\alpha_6 \neq 0$, then by choosing $x = \frac{\alpha_2}{\alpha_6}, r = -\frac{\alpha_2^2\alpha_4}{\alpha_6^3}, s = 0, t = -\frac{\alpha_1}{2\alpha_6}$, we have the representative $\langle \nabla_2 + \alpha\nabla_3 + \nabla_6 \rangle$.
2. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 \neq 0$ then we have
   (a) if $\alpha_5 \neq -\alpha_6$, then we have the following subcases:
      (i) if $\alpha_2 = 0, \alpha_1 = 0$, then by choosing
      $$x = 2\alpha_5(\alpha_5 + \alpha_6),\ s = 2\alpha_5(\alpha_4^2(2\alpha_5 + \alpha_6) - \alpha_3(\alpha_5 + \alpha_6)^2), \\ r = -4\alpha_4\alpha_5^2(\alpha_5 + \alpha_6),$$

we have the family of representatives $\langle \nabla_5 + \alpha\nabla_6 \rangle_{\alpha\neq -1}$;

(ii) if $\alpha_2 = 0, \alpha_1 \neq 0$, then by choosing

$$x = \sqrt[4]{\frac{\alpha_1}{\alpha_5}},\ r = -\frac{\alpha_4\sqrt{\alpha_1}}{(\alpha_5+\alpha_6)\sqrt{\alpha_5}},$$

$$s = \frac{((\alpha_5+\alpha_6)(2\alpha_4^2-\alpha_2\alpha_4)-\alpha_3(\alpha_5+\alpha_6)^2-\alpha_4^2\alpha_6)\sqrt{\alpha_1}}{2\alpha_5(\alpha_5+\alpha_6)^2\sqrt{\alpha_5}}, t = 0,$$

we have the family of representatives $\langle \nabla_1 + \nabla_5 + \alpha\nabla_6 \rangle_{\alpha\neq -1}$;

(iii) if $\alpha_2 \neq 0$, then by choosing

$$x = \frac{\alpha_2}{\alpha_5},\ r = -\frac{\alpha_2^2\alpha_4}{\alpha_5^2(\alpha_5+\alpha_6)},$$

$$s = \frac{\alpha_2^2((\alpha_5+\alpha_6)(2\alpha_4^2-\alpha_2\alpha_4)-\alpha_3(\alpha_5+\alpha_6)^2-\alpha_4^2\alpha_6)}{2\alpha_5^3(\alpha_5+\alpha_6)^2}, t = -\frac{\alpha_1}{2\alpha_5},$$

we have the family of representatives $\langle \nabla_2 + \nabla_5 + \alpha\nabla_6 \rangle_{\alpha\neq -1}$.

(b) if $\alpha_6 = -\alpha_5$, then we have the following subcases:

(i) if $\alpha_4 = 0, \alpha_2 = 0, \alpha_1 = 0$, then we have the representative $\langle \nabla_5 - \nabla_6 \rangle$, which will be jointed with the family from the case (2(a)i);

(ii) if $\alpha_4 = 0, \alpha_2 = 0, \alpha_1 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_1}{\alpha_5}}, r = 0, s = -\frac{\alpha_3\sqrt{\alpha_1}}{2\alpha_5\sqrt{\alpha_4}}, t = 0$, we have the representative $\langle \nabla_1 + \nabla_5 - \nabla_6 \rangle$, which will be jointed with the family from the case (2(a)ii);

(iii) if $\alpha_4 = 0, \alpha_2 \neq 0$, then by choosing $x = \frac{\alpha_2}{\alpha_5}, r = 0, s = -\frac{\alpha_2^2\alpha_3}{2\alpha_5^3}, t = -\frac{\alpha_1}{2\alpha_5}$, we have the representative $\langle \nabla_2 + \nabla_5 - \nabla_6 \rangle$, which will be jointed with the family from the case (2(a)iii);

(iv) if $\alpha_4 \neq 0$, then by choosing $x = \frac{\alpha_4}{\alpha_5}$, $s = -\frac{\alpha_3\alpha_4^2}{2\alpha_5^3}$, $r = 0$, we have the families of representatives

$$\langle \alpha\nabla_1 + \nabla_4 + \nabla_5 - \nabla_6 \rangle \text{ and } \langle \alpha\nabla_2 + \nabla_4 + \nabla_5 - \nabla_6 \rangle_{\alpha\neq 0}$$

depending on $\alpha_2 = 0$ or not.

3. $\alpha_8 = 0, \alpha_7 \neq 0$, then by choosing

$$r = -\frac{\alpha_5}{\alpha_7}x^2, s = \frac{\alpha_5\alpha_6-\alpha_4\alpha_7}{\alpha_7^2}x^2, t = \frac{\alpha_3\alpha_7^2-2\alpha_4\alpha_5\alpha_7+\alpha_5^2\alpha_6+\alpha_2\alpha_5\alpha_7}{\alpha_7^3}x,$$

we have $\alpha_3^* = \alpha_4^* = \alpha_5^* = 0$. Therefore, we can suppose that $\alpha_3 = 0, \alpha_4 = 0, \alpha_5 = 0$, and we have

(a) if $\alpha_1 = 0, \alpha_2 = 0$, then we have the representatives $\langle \nabla_7 \rangle$ and $\langle \nabla_6 + \nabla_7 \rangle$ depending on whether $\alpha_6 = 0$ or not;

(b) if $\alpha_1 = 0, \alpha_2 \neq 0$, then by choosing $x = \sqrt{\alpha_2\alpha_7^{-1}}, r = 0, s = 0, t = 0$, we have the family of representative $\langle \nabla_2 + \alpha\nabla_6 + \nabla_7 \rangle$;

(c) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[5]{\frac{\alpha_1}{\alpha_7}}, r = 0, s = 0, t = 0$, we have the family of representative $\langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_6 + \nabla_7 \rangle$.

4. $\alpha_8 \neq 0$, then by choosing $r = -\frac{\alpha_7}{\alpha_8}x^2, t = -\frac{\alpha_2}{\alpha_8}x, s = -\frac{\alpha_5x^2+\alpha_7r}{\alpha_8}$ we have $\alpha_2^* = \alpha_5^* = \alpha_7^* = 0$. Therefore, we can suppose that $\alpha_2 = 0, \alpha_5 = 0, \alpha_7 = 0$, then we have

(a) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0$, then we have the representatives $\langle \nabla_8 \rangle$ and $\langle \nabla_6 + \nabla_8 \rangle$ depending on whether $\alpha_6 = 0$ or not;

(b) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_4\alpha_8^{-1}}, r = 0, s = 0, t = 0$, we have the family of representative $\langle \nabla_4 + \alpha\nabla_6 + \nabla_8 \rangle$;

(c) if $\alpha_1 = 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_3\alpha_8^{-1}}, r = 0, s = 0, t = 0$, we have the family of representative $\langle \nabla_3 + \alpha\nabla_4 + \beta\nabla_6 + \nabla_8 \rangle$;

(d) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[6]{\alpha_1\alpha_8^{-1}}, r = 0, s = 0, t = 0$, we have the family of representative $\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \gamma\nabla_6 + \nabla_8 \rangle$.

Summarizing all cases we have the following distinct orbits:

$$\langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_6 + \nabla_7 \rangle^{O(\alpha,\beta)=O(\eta_5^2\alpha,-\eta_5\beta)=O(\eta_5^4\alpha,\eta_5^2\beta)=O(-\eta_5\alpha,-\eta_5^3\beta)=O(-\eta_5^3\alpha,\eta_5^4\beta)},$$
$$\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \gamma\nabla_6 + \nabla_8 \rangle^{\substack{O(\alpha,\beta,\gamma)=O(-\eta_3\alpha,\beta,\eta_3^2\gamma)=O(-\eta_3\alpha,-\beta,\eta_3^2\gamma)=\\ O(\eta_3^2\alpha,-\beta,-\eta_3\gamma)=O(\eta_3^2\alpha,\beta,-\eta_3\gamma)=O(\alpha,-\beta,\gamma)}},$$
$$\langle \alpha\nabla_1 + \nabla_4 + \nabla_5 - \nabla_6 \rangle, \langle \nabla_1 + \nabla_5 + \alpha\nabla_6 \rangle, \langle 2\nabla_2 + \nabla_3 + \nabla_4 \rangle, \langle \nabla_2 + \alpha\nabla_3 + \nabla_6 \rangle,$$
$$\langle \nabla_2 + \alpha\nabla_4 \rangle, \langle \alpha\nabla_2 + \nabla_4 + \nabla_5 - \nabla_6 \rangle_{\alpha\neq 0}, \langle \nabla_2 + \nabla_5 + \alpha\nabla_6 \rangle,$$
$$\langle \nabla_2 + \alpha\nabla_6 + \nabla_7 \rangle^{O(\alpha)=O(-\alpha)},$$
$$\langle \nabla_3 + \alpha\nabla_4 + \beta\nabla_6 + \nabla_8 \rangle^{O(\alpha,\beta)=O(i\alpha,-\beta)=O(-i\alpha,-\beta)=O(-\alpha,\beta)},$$
$$\langle \nabla_4 + \alpha\nabla_6 + \nabla_8 \rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)}, \langle \nabla_5 + \alpha\nabla_6 \rangle, \langle \nabla_6 + \nabla_7 \rangle, \langle \nabla_6 + \nabla_8 \rangle,$$
$$\langle \nabla_7 \rangle, \langle \nabla_8 \rangle,$$

which gives the following new algebras:

$\mathbf{N}_{215}^{\alpha,\beta}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	
		$e_2e_2 = e_4$	$e_3e_3 = \beta e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{216}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4 + \alpha e_5$	
		$e_2e_3 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{217}^{\alpha}$	:	$e_1e_1 = \alpha e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = -e_5$		
$\mathbf{N}_{218}^{\alpha}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$		
		$e_2e_2 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{219}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = 2e_5$	$e_2e_2 = e_4 + e_5$	$e_2e_3 = e_5$
$\mathbf{N}_{220}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_2 = e_4 + \alpha e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{221}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_2 = e_4$	$e_2e_3 = \alpha e_5$
$\mathbf{N}_{222}^{\alpha\neq 0}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_4$	
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = -e_5$		
$\mathbf{N}_{223}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$		
		$e_2e_2 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{224}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$		
		$e_2e_2 = e_4$	$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{225}^{\alpha,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4 + e_5$		
		$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{226}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$		
		$e_2e_3 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{227}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$
$\mathbf{N}_{228}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_3e_3 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{229}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_3e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{230}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{231}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_4e_4 = e_5$	

3.11. **1-dimensional central extensions of $\mathbf{N}_{16}^{4*}$.** Here we will collect all information about $\mathbf{N}_{16}^{4*}$:

$\mathbf{N}_{16}^{4*}$	$e_1e_2 = e_3$ $e_1e_3 = e_4$ $e_2e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{16}^{4*}) = \langle [\Delta_{11}], [\Delta_{22}], [\Delta_{23}], [\Delta_{33}] \rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{16}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{16}^{4*}) \oplus \langle [\Delta_{14}], [\Delta_{24}], [\Delta_{34}], [\Delta_{44}] \rangle$

$$\phi_1 = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & x & 0 & 0 \\ 0 & 0 & x^2 & 0 \\ t & s & 0 & x^3 \end{pmatrix}, \phi_2 = \begin{pmatrix} 0 & y & 0 & 0 \\ y & 0 & 0 & 0 \\ 0 & 0 & y^2 & 0 \\ t & s & 0 & y^3 \end{pmatrix}$$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{11}], & \nabla_2 = [\Delta_{14}], & \nabla_3 = [\Delta_{22}], & \nabla_4 = [\Delta_{23}], \\ \nabla_5 = [\Delta_{24}], & \nabla_6 = [\Delta_{33}], & \nabla_7 = [\Delta_{34}], & \nabla_8 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^{4*}_{16})$. Since

$$\phi^T \begin{pmatrix} \alpha_1 & 0 & 0 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 & \alpha_5 \\ 0 & \alpha_4 & \alpha_6 & \alpha_7 \\ \alpha_2 & \alpha_5 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha_1^* & \alpha^* & \alpha^{**} & \alpha_2^* \\ \alpha^* & \alpha_3^* & \alpha_4^* + \alpha^{**} & \alpha_5^* \\ \alpha^{**} & \alpha_4^* + \alpha^{**} & \alpha_6^* & \alpha_7^* \\ \alpha_2^* & \alpha_5^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

in the case $\phi = \phi_1$, we have

$$\begin{array}{ll} \alpha_1^* = \alpha_1 x^2 + 2\alpha_2 xt + \alpha_8 t^2, & \alpha_2^* = (\alpha_2 x + \alpha_8 t)x^3, \\ \alpha_3^* = \alpha_3 x^2 + 2\alpha_5 xs + \alpha_8 s^2, & \alpha_4^* = (\alpha_4 x + \alpha_7 s)x^2 - \alpha_7 x^2 t, \\ \alpha_5^* = (\alpha_5 x + \alpha_8 s)x^3, & \alpha_6^* = \alpha_6 x^4, \\ \alpha_7^* = \alpha_7 x^5, & \alpha_8^* = \alpha_8 x^6; \end{array}$$

and on the opposite case, for $\phi = \phi_2$, we have

$$\begin{array}{ll} \alpha_1^* = \alpha_3 y^2 + 2\alpha_5 ty + \alpha_8 t^2, & \alpha_2^* = (\alpha_5 y + \alpha_8 t)y^3, \\ \alpha_3^* = \alpha_1 y^2 + 2\alpha_2 sy + \alpha_8 s^2, & \alpha_4^* = ((s-t)\alpha_7 - y\alpha_4)y^2, \\ \alpha_5^* = (y\alpha_2 + s\alpha_8)y^3, & \alpha_6^* = \alpha_6 y^4, \\ \alpha_7^* = \alpha_7 y^5, & \alpha_8^* = \alpha_8 y^6. \end{array}$$

We are interested in $(\alpha_2, \alpha_5, \alpha_7, \alpha_8) \neq (0,0,0,0)$. Let us consider the following cases:

1. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 = 0$, then $\alpha_2 \neq 0$ and
   (a) if $\alpha_4 \neq 0$, then by choosing $\phi = \phi_1$, $x = \alpha_4 \alpha_2^{-1}$, $t = -\frac{\alpha_1 \alpha_4}{2\alpha_2^2}$, we have the family of representatives $\langle \nabla_2 + \alpha \nabla_3 + \nabla_4 + \beta \nabla_6 \rangle$;
   (b) if $\alpha_4 = 0, \alpha_3 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt{\alpha_3 \alpha_2^{-1}}$, $t = -\frac{\alpha_1 \sqrt{\alpha_3}}{2\sqrt{\alpha_2^3}}$, we have the family of representatives $\langle \nabla_2 + \nabla_3 + \alpha \nabla_6 \rangle$;
   (c) if $\alpha_4 = 0, \alpha_3 = 0$, then by choosing $\phi = \phi_1$, $x = 2\alpha_2$, $t = -\alpha_1$, $s = 0$, we have the family of representatives $\langle \nabla_2 + \alpha \nabla_6 \rangle$.
2. $\alpha_8 = 0, \alpha_7 = 0, \alpha_5 \neq 0$ and
   (a) if $\alpha_2 \neq 0, \alpha_4 \neq 0$, then by choosing
   $$\phi = \phi_1,\ x = \tfrac{\alpha_4}{\alpha_5},\ t = -\tfrac{\alpha_1 \alpha_4}{2\alpha_2 \alpha_5},\ s = -\tfrac{\alpha_3 \alpha_4}{2\alpha_5^2},$$
   we have the following family of representatives
   $$\langle \alpha \nabla_2 + \nabla_4 + \nabla_5 + \beta \nabla_6 \rangle_{\alpha \neq 0};$$
   (b) if $\alpha_2 \neq 0, \alpha_4 = 0$, then by choosing
   $$\phi = \phi_1,\ x = 2\alpha_2 \alpha_5,\ t = -\alpha_1 \alpha_5,\ s = -\alpha_2 \alpha_3,$$
   we have the following family of representatives $\langle \alpha \nabla_2 + \nabla_5 + \beta \nabla_6 \rangle_{\alpha \neq 0}$;

   (c) if $\alpha_2 = 0$, then by choosing $\phi = \phi_2$, $y = 1$, $t = 0$, $s = 0$, we have the representative with $\alpha_5^* = 0$ and $\alpha_2^* \neq 0$, which was considered above.
3. $\alpha_8 = 0, \alpha_7 \neq 0$, then we have
   (a) if $\alpha_2 = 0, \alpha_5 = 0, \alpha_1 = 0, \alpha_3 = 0$, then we have the representatives $\langle \nabla_7 \rangle$ and $\langle \nabla_6 + \nabla_7 \rangle$ depending on whether $\alpha_6 = 0$ or not;
   (b) if $\alpha_2 = 0, \alpha_5 = 0, \alpha_1 \neq 0$, then by choosing
$$\phi = \phi_1,\ x = \sqrt[3]{\alpha_1 \alpha_7^{-1}}, s = 0, t = \alpha_4 \sqrt[3]{\alpha_1} \alpha_7^{-1},$$
   we have the family of representatives $\langle \nabla_1 + \alpha \nabla_3 + \beta \nabla_6 + \nabla_7 \rangle$;
   (c) if $\alpha_2 \neq 0$, then by choosing
$$\phi = \phi_1,\ x = \alpha_2 \alpha_7^{-1}, s = -(\alpha_1 \alpha_7 + 2\alpha_2 \alpha_4)/(2\alpha_7^2), t = -\alpha_1/(2\alpha_7),$$
   we have the family of representatives $\langle \nabla_2 + \alpha \nabla_3 + \beta \nabla_5 + \gamma \nabla_6 + \nabla_7 \rangle$.
4. $\alpha_8 \neq 0$, then by choosing $\phi = \phi_1$, $t = -\frac{\alpha_2}{\alpha_8} x, s = -\frac{\alpha_5}{\alpha_8} x$, we have $\alpha_2^* = \alpha_5^* = 0$. Now we can suppose that $\alpha_2 = 0, \alpha_5 = 0$ and we have
   (a) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_6 = 0$, then we have the representatives $\langle \nabla_8 \rangle$ and $\langle \nabla_7 + \nabla_8 \rangle$ depending on whether $\alpha_7 = 0$ or not;
   (b) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_6 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt{\alpha_6 \alpha_8^{-1}}$, $s = 0$, $t = 0$, we have the family of representative $\langle \nabla_6 + \alpha \nabla_7 + \nabla_8 \rangle$;
   (c) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt[3]{\alpha_4 \alpha_8^{-1}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_4 + \alpha \nabla_6 + \beta \nabla_7 + \nabla_8 \rangle$;
   (d) if $\alpha_1 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt[4]{\alpha_1 \alpha_8^{-1}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha \nabla_3 + \beta \nabla_4 + \gamma \nabla_6 + \mu \nabla_7 + \nabla_8 \rangle$.

Summarizing, we have the following distinct orbits:

$$\langle \nabla_1 + \alpha \nabla_3 + \beta \nabla_4 + \gamma \nabla_6 + \mu \nabla_7 + \nabla_8 \rangle^{\begin{array}{l} O(\alpha, \beta, \gamma, \mu) = O(\alpha, i\beta, -\gamma, -i\mu) = \\ O(\alpha, -i\beta, -\gamma, i\mu) = O(\alpha, -\beta, \gamma, -\mu) = \\ O(\frac{1}{\alpha}, -\frac{\beta}{\sqrt[4]{\alpha^3}}, \frac{\gamma}{\sqrt{\alpha}}, \frac{\mu}{\sqrt[4]{\alpha}}) = \\ O(\frac{1}{\alpha}, -\frac{i\beta}{\sqrt[4]{\alpha^3}}, -\frac{\gamma}{\sqrt{\alpha}}, -\frac{i\mu}{\sqrt[4]{\alpha}}) = \\ O(\frac{1}{\alpha}, \frac{\beta}{\sqrt[4]{\alpha^3}}, \frac{\gamma}{\sqrt{\alpha}}, -\frac{\mu}{\sqrt[4]{\alpha}}) = O(\frac{1}{\alpha}, -\frac{\beta}{\sqrt[4]{\alpha^3}}, \frac{\gamma}{\sqrt{\alpha}}, \frac{\mu}{\sqrt[4]{\alpha}}) \end{array}},$$
$$\langle \nabla_1 + \alpha \nabla_3 + \beta \nabla_6 + \nabla_7 \rangle^{\begin{array}{l} O(\alpha, \beta) = O(\alpha, -\eta_3 \beta) = O(\alpha, \eta_3^2 \beta) = \\ O(\alpha^{-1}, -\eta_3 \beta \sqrt[3]{\alpha^{-1}}) = O(\alpha^{-1}, \eta_3^2 \beta \sqrt[3]{\alpha^{-1}}) = O(\alpha^{-1}, \beta \sqrt[3]{\alpha^{-1}}) \end{array}},$$
$$\langle \nabla_2 + \alpha \nabla_3 + \nabla_4 + \beta \nabla_6 \rangle, \langle \nabla_2 + \alpha \nabla_3 + \beta \nabla_5 + \gamma \nabla_6 + \nabla_7 \rangle^{O(\alpha,\beta,\gamma)=O(-\frac{\alpha}{\beta^4}, \frac{1}{\beta}, \frac{\gamma}{\beta})},$$
$$\langle \nabla_2 + \nabla_3 + \alpha \nabla_6 \rangle, \langle \alpha \nabla_2 + \nabla_4 + \nabla_5 + \beta \nabla_6 \rangle^{O(\alpha,\beta)=O(\alpha^{-1},\beta\alpha^{-1})}_{\alpha \neq 0},$$
$$\langle \alpha \nabla_2 + \nabla_5 + \beta \nabla_6 \rangle^{O(\alpha,\beta)=O(\alpha^{-1},\beta\alpha^{-1})}_{\alpha \neq 0}, \langle \nabla_2 + \alpha \nabla_6 \rangle,$$
$$\langle \nabla_4 + \alpha \nabla_6 + \beta \nabla_7 + \nabla_8 \rangle^{\begin{array}{l} O(\alpha, \beta) = O(\eta_3^2 \alpha, -\eta_3 \beta) = O(-\eta_3 \alpha, \eta_3^2 \beta) = \\ O(\eta_3^2 \alpha, \eta_3 \beta) = O(-\eta_3 \alpha, -\eta_3^2 \beta) = O(\alpha, -\beta) \end{array}}, \langle \nabla_6 + \nabla_7 \rangle,$$
$$\langle \nabla_6 + \alpha \nabla_7 + \nabla_8 \rangle^{O(\alpha)=O(-\alpha)}, \langle \nabla_7 \rangle, \langle \nabla_7 + \nabla_8 \rangle, \langle \nabla_8 \rangle,$$

which gives the following new algebras:

$\mathbf{N}_{232}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4 + \beta e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = \mu e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{233}^{\alpha,\beta}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4$	$e_3e_3 = \beta e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{234}^{\alpha,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = \alpha e_5$	$e_2e_3 = e_4 + e_5$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{235}^{\alpha,\beta,\gamma}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4$	$e_2e_4 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{236}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	

		$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{237}^{\alpha\neq 0,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	
		$e_2e_3 = e_4 + e_5$	$e_2e_4 = e_5$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{238}^{\alpha\neq 0,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	
		$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{239}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_3 = e_4$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{240}^{\alpha,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4 + e_5$	
		$e_3e_3 = \alpha e_5$	$e_3e_4 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{241}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	
		$e_3e_3 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{242}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	
		$e_3e_3 = e_5$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{243}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{244}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	
		$e_3e_4 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{245}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	$e_4e_4 = e_5$

3.12. **1-dimensional central extensions of $\mathbf{N}_{17}^{4*}$.** Here we will collect all information about $\mathbf{N}_{17}^{4*}$:

$\mathbf{N}_{17}^{4*}$	$e_1e_2 = e_3$ $e_3e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{17}^{4*}) = \langle [\Delta_{11}], [\Delta_{13}], [\Delta_{22}], [\Delta_{23}] \rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{17}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{17}^{4*}) \oplus \langle [\Delta_{14}], [\Delta_{24}], [\Delta_{34}], [\Delta_{44}] \rangle$
$\phi_1 = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & xq & 0 \\ t & s & 0 & x^2q^2 \end{pmatrix}, \phi_2 = \begin{pmatrix} 0 & p & 0 & 0 \\ y & 0 & 0 & 0 \\ 0 & 0 & yp & 0 \\ t & s & 0 & y^2p^2 \end{pmatrix}$		

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{11}], & \nabla_2 = [\Delta_{13}], & \nabla_3 = [\Delta_{14}], & \nabla_4 = [\Delta_{22}], \\ \nabla_5 = [\Delta_{23}], & \nabla_6 = [\Delta_{24}], & \nabla_7 = [\Delta_{34}], & \nabla_8 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{17}^{4*})$. Since

$$\phi^T \begin{pmatrix} \alpha_1 & 0 & \alpha_2 & \alpha_3 \\ 0 & \alpha_4 & \alpha_5 & \alpha_6 \\ \alpha_2 & \alpha_5 & 0 & \alpha_7 \\ \alpha_3 & \alpha_6 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha_1^* & \alpha^* & \alpha_2^* & \alpha_3^* \\ \alpha^* & \alpha_4^* & \alpha_5^* & \alpha_6^* \\ \alpha_2^* & \alpha_5^* & 0 & \alpha_7^* \\ \alpha_3^* & \alpha_6^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

then in the case $\phi = \phi_1$, we have

$$\begin{array}{ll} \alpha_1^* = \alpha_1 x^2 + 2\alpha_3 xt + \alpha_8 t^2, & \alpha_2^* = (\alpha_2 x + \alpha_7 t)xq, \\ \alpha_3^* = (\alpha_3 x + \alpha_8 t)x^2q^2, & \alpha_4^* = \alpha_4 q^2 + 2\alpha_6 qs + \alpha_8 s^2, \\ \alpha_5^* = (\alpha_5 q + \alpha_7 s)xq, & \alpha_6^* = (\alpha_6 q + \alpha_8 s)x^2q^2, \\ \alpha_7^* = \alpha_7 x^3q^3, & \alpha_8^* = \alpha_8 x^4q^4; \end{array}$$

and in the opposite case $\phi = \phi_2$, we have

$$\begin{array}{ll} \alpha_1^* = \alpha_4 p^2 + 2\alpha_6 pt + \alpha_8 t^2, & \alpha_2^* = (\alpha_5 p + \alpha_7 t)py, \\ \alpha_3^* = (\alpha_6 p + \alpha_8 t)p^2y^2, & \alpha_4^* = \alpha_1 y^2 + 2\alpha_3 sy + \alpha_8 s^2, \\ \alpha_5^* = (\alpha_2 y + \alpha_7 s)py, & \alpha_6^* = (\alpha_3 y + \alpha_8 s)p^2y^2, \end{array}$$

$$\alpha_7^* = \alpha_7 p^3 y^3, \qquad \alpha_8^* = \alpha_8 p^4 y^4.$$

We are interested in $(\alpha_3, \alpha_6, \alpha_7, \alpha_8) \neq (0,0,0,0)$. Let us consider the following cases:

1. $\alpha_8 = 0, \alpha_7 = 0, \alpha_6 = 0$, then $\alpha_3 \neq 0$ and choosing $\phi = \phi_1$, $t = -\frac{\alpha_1}{2\alpha_3}x$, we get $\alpha_1^* = 0$. Now consider the following subcases:
   (a) if $\alpha_2 = 0, \alpha_4 = 0, \alpha_5 = 0$, then we have the representative $\langle \nabla_3 \rangle$;
   (b) if $\alpha_2 = 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt{\frac{\alpha_5}{\alpha_3}}, q = 1, s = 0, t = -\frac{\alpha_1\sqrt{\alpha_5}}{2\alpha_3\sqrt{\alpha_3}}$, we have the representative $\langle \nabla_3 + \nabla_5 \rangle$;
   (c) if $\alpha_2 = 0, \alpha_4 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt[3]{\frac{\alpha_4}{\alpha_3}}, q = 1, s = 0, t = -\frac{\alpha_1\sqrt[3]{\alpha_5}}{2\alpha_3\sqrt[3]{\alpha_3}}$, we have the representative $\langle \nabla_3 + \nabla_4 + \alpha\nabla_5 \rangle$;
   (d) if $\alpha_2 \neq 0, \alpha_4 = 0, \alpha_5 = 0$, then by choosing $\phi = \phi_1$, $x = \alpha_2, q = \frac{1}{\alpha_3}, s = 0, t = -\frac{\alpha_1\alpha_2}{2\alpha_3}$, we have the representative $\langle \nabla_2 + \nabla_3 \rangle$;
   (e) if $\alpha_2 \neq 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt{\frac{\alpha_5}{\alpha_3}}, q = \sqrt{\frac{\alpha_2^2}{\alpha_3\alpha_5}}, s = 0, t = -\frac{\alpha_1\sqrt{\alpha_5}}{2\alpha_3\sqrt{\alpha_3}}$, we have the representative $\langle \nabla_2 + \nabla_3 + \nabla_5 \rangle$;
   (f) if $\alpha_2 \neq 0, \alpha_4 \neq 0$, then by choosing $\phi = \phi_1$, $x = \sqrt[3]{\alpha_4\alpha_3^{-3}}, q = \sqrt[3]{\alpha_2^3\alpha_3^{-2}\alpha_4^{-1}}$, we have the family of representative $\langle \nabla_2 + \nabla_3 + \nabla_4 + \alpha\nabla_5 \rangle$.
2. $\alpha_8 = 0, \alpha_7 = 0, \alpha_6 \neq 0$, and $\alpha_3 = 0$, then by choosing some suitable automorphism ϕ_2 we have $\alpha_3^* \neq 0$ which is the case considered above. Now we can suppose that $\alpha_3 \neq 0$, and choosing $t = -\frac{\alpha_1}{2\alpha_3}x, s = -\frac{\alpha_4}{2\alpha_6}x$, we have $\alpha_1^* = 0, \alpha_4^* = 0$. Therefore, we can suppose that $\alpha_1 = 0, \alpha_4 = 0$. Consider the following subcases:
   (a) $\alpha_2 = 0, \alpha_5 = 0$, then by choosing $\phi = \phi_1$, $x = \alpha_6, q = \alpha_3, s = 0, t = 0$, we have the representative $\langle \nabla_3 + \nabla_6 \rangle$;
   (b) $\alpha_2 \neq 0$, then by choosing $\phi = \phi_1, x = \alpha_3^{-1}\sqrt{\alpha_2\alpha_6}, q = \sqrt{\alpha_2\alpha_6^{-1}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_2 + \nabla_3 + \alpha\nabla_5 + \nabla_6 \rangle$.
3. $\alpha_8 = 0, \alpha_7 \neq 0$, then by choosing $\phi = \phi_1$, $t = -\alpha_2\alpha_7^{-1}x, s = -\alpha_5\alpha_7^{-1}q$, we have $\alpha_2^* = 0, \alpha_5^* = 0$. Therefore, we can suppose that $\alpha_2 = 0, \alpha_5 = 0$. Consider the following subcases:
   (a) if $\alpha_1 = 0, \alpha_4 = 0, \alpha_3 = 0, \alpha_6 = 0$, then we have the representative $\langle \nabla_7 \rangle$;
   (b) if $\alpha_1 = 0, \alpha_4 = 0, \alpha_3 = 0, \alpha_6 \neq 0$, then by choosing $\phi = \phi_1$, $x = \alpha_6\alpha_7^{-1}, q = 1, s = 0, t = 0$, we have the representative $\langle \nabla_6 + \nabla_7 \rangle$;
   (c) if $\alpha_1 = 0, \alpha_4 = 0, \alpha_3 \neq 0$, and $\alpha_6 = 0$, then by choosing some suitable automorphism ϕ_2, we have $\alpha_6^* \neq 0$. Thus we can consider the case $\alpha_6 \neq 0$ and choosing $\phi = \phi_1$, $x = \alpha_6\alpha_7^{-1}, q = \alpha_3\alpha_7^{-1}, s = 0, t = 0$, we have the representative $\langle \nabla_3 + \nabla_6 + \nabla_7 \rangle$;
   (d) if $\alpha_1 = 0, \alpha_4 \neq 0, \alpha_3 = 0, \alpha_6 = 0$, then by choosing $\phi = \phi_1$, $x = 1, q = \alpha_4\alpha_7^{-1}, s = 0, t = 0$, we have the representative $\langle \nabla_4 + \nabla_7 \rangle$;
   (e) if $\alpha_1 = 0, \alpha_4 \neq 0, \alpha_3 = 0, \alpha_6 \neq 0$, then by choosing $\phi = \phi_1$, $x = \alpha_6\alpha_7{}^{-1}, q = \alpha_4\alpha_7^2\alpha_6^{-3}$, we have the representative $\langle \nabla_4 + \nabla_6 + \nabla_7 \rangle$;
   (f) if $\alpha_1 = 0, \alpha_4 \neq 0, \alpha_3 \neq 0$, then by choosing
   $$\phi = \phi_1,\ x = \sqrt[3]{\alpha_4\alpha_3^{-1}}, q = \alpha_3\alpha_7^{-1}, s = 0, t = 0,$$
   we have the family of representatives $\langle \nabla_3 + \nabla_4 + \alpha\nabla_6 + \nabla_7 \rangle$;
   (g) if $\alpha_1 \neq 0$. In case of $\alpha_4 = 0$, choosing some suitable automorphism ϕ_2, we have $\alpha_4^* \neq 0$. Thus, we can suppose $\alpha_4 \neq 0$, and choosing

$$\phi=\phi_1,\ x=\sqrt[8]{\alpha_4^3\alpha_1^{-1}\alpha_7^{-2}},q=\sqrt[8]{\alpha_1^3\alpha_4^{-1}\alpha_7^{-2}},s=0,t=0,$$
we have the family of representatives $\langle\nabla_1+\alpha\nabla_3+\nabla_4+\beta\nabla_6+\nabla_7\rangle$.

4. $\alpha_8\neq 0$, then by choosing $\phi=\phi_1$, $t=-\frac{\alpha_3}{\alpha_8}x, s=-\frac{\alpha_6}{\alpha_8}q$, we have $\alpha_3^*=0,\alpha_6^*=0$. Consider the following cases:
   (a) if $\alpha_1=0,\alpha_4=0,\alpha_2=0,\alpha_5=0$, then we have the representatives $\langle\nabla_8\rangle$ and $\langle\nabla_7+\nabla_8\rangle$ depending on whether $\alpha_7=0$ or not;
   (b) if $\alpha_1=0,\alpha_4=0,\alpha_2=0,\alpha_5\neq 0$, then we have the representatives $\langle\nabla_5+\nabla_8\rangle$ and $\langle\nabla_5+\nabla_7+\nabla_8\rangle$ depending on whether $\alpha_7=0$ or not;
   (c) if $\alpha_1=0,\alpha_4=0,\alpha_2\neq 0$. In case of $\alpha_5=0$, choosing some suitable automorphism ϕ_2, we have $\alpha_5^*\neq 0$. Thus, we can suppose $\alpha_5\neq 0$, and choosing
   $$\phi=\phi_1,\ x=\sqrt[5]{\alpha_5^3\alpha_2^{-2}\alpha_8^{-1}},q=\sqrt[5]{\alpha_2^3\alpha_5^{-2}\alpha_8^{-1}},s=0,t=0,$$
   we have the family of representatives $\langle\nabla_2+\nabla_5+\alpha\nabla_7+\nabla_8\rangle$;
   (d) if $\alpha_1=0,\alpha_4\neq 0,\alpha_2=0,\alpha_5=0$, then we have the representatives $\langle\nabla_4+\nabla_8\rangle$ and $\langle\nabla_4+\nabla_7+\nabla_8\rangle$ depending on whether $\alpha_7=0$ or not;
   (e) if $\alpha_1=0,\alpha_4\neq 0,\alpha_2=0,\alpha_5\neq 0$, then by choosing
   $$\phi=\phi_1,\ x=\alpha_4\alpha_5^{-1},q=\alpha_5^2\alpha_4^{-1}\sqrt{\alpha_4^{-1}\alpha_8^{-1}},s=0,t=0,$$
   we have the family of representatives $\langle\nabla_4+\nabla_5+\alpha\nabla_7+\nabla_8\rangle$;
   (f) if $\alpha_1=0,\alpha_4\neq 0,\alpha_2\neq 0$, then by choosing
   $$\phi=\phi_1,\ x=\sqrt[8]{\alpha_4^3\alpha_2^{-2}\alpha_8^{-1}},q=\sqrt[4]{\alpha_2^2\alpha_4^{-1}\alpha_8^{-1}},s=0,t=0,$$
   we have the family of representatives $\langle\nabla_2+\nabla_4+\alpha\nabla_5+\beta\nabla_7+\nabla_8\rangle$;
   (g) if $\alpha_1\neq 0$ then by choosing some suitable automorphism ϕ_2, we have $\alpha_4^*\neq 0$. Thus, we can suppose $\alpha_4\neq 0$, and choosing
   $$\phi=\phi_1,\ x=\sqrt[6]{\alpha_4^2\alpha_1^{-1}\alpha_8^{-1}},q=\sqrt[6]{\alpha_1^2\alpha_4^{-1}\alpha_8^{-1}},s=0,t=0,$$
   we have the family of representatives
   $$\langle\nabla_1+\alpha\nabla_2+\nabla_4+\beta\nabla_5+\gamma\nabla_7+\nabla_8\rangle.$$

Summarizing, we have the following distinct orbits:

$$\langle\nabla_1+\alpha\nabla_2+\nabla_4+\beta\nabla_5+\gamma\nabla_7+\nabla_8\rangle^{\begin{array}{l}O(\alpha,\beta,\gamma)=O(\eta_3^2\alpha,\eta_3^2\beta,\eta_3^2\gamma)=O(-\eta_3^2\alpha,\eta_3^2\beta,-\eta_3^2\gamma)=\\ O(\eta_3^2\alpha,-\eta_3^2\beta,-\eta_3^2\gamma)=O(-\eta_3^2\alpha,\eta_3^2\beta,\eta_3^2\gamma)=\\ O(\eta_3\alpha,\eta_3\beta,-\eta_3\gamma)=O(-\eta_3\alpha,\eta_3\beta,\eta_3\gamma)=\\ O(\eta_3\alpha,-\eta_3\beta,\eta_3\gamma)=O(-\eta_3\alpha,-\eta_3\beta,-\eta_3\gamma)=\\ O(-\alpha,\beta,-\gamma)=O(\alpha,-\beta,-\gamma)=\\ O(-\alpha,-\beta,\gamma)=O(\beta,\alpha,\gamma)=\\ O(\eta_3^2\beta,\eta_3^2\alpha,\eta_3^2\gamma)=O(-\eta_3^2\beta,\eta_3^2\alpha,-\eta_3^2\gamma)=\\ O(\eta_3^2\beta,-\eta_3^2\alpha,-\eta_3^2\gamma)=O(-\eta_3^2\beta,\eta_3^2\alpha,\eta_3^2\gamma)=\\ O(\eta_3\beta,\eta_3\alpha,-\eta_3\gamma)=O(-\eta_3\beta,\eta_3\alpha,\eta_3\gamma)=\\ O(\eta_3\beta,-\eta_3\alpha,\eta_3\gamma)=O(-\eta_3\beta,-\eta_3\alpha,-\eta_3\gamma)=\\ O(-\beta,\alpha,-\gamma)=O(\beta,-\alpha,-\gamma)=O(-\beta,-\alpha,\gamma)\end{array}},$$

$$\langle\nabla_1+\alpha\nabla_3+\nabla_4+\beta\nabla_6+\nabla_7\rangle^{\begin{array}{l}O(\alpha,\beta)=O(\eta_4\alpha,-\eta_4\beta)=O(-\eta_4\alpha,\eta_4\beta)=O(\eta_4^3\alpha,-\eta_4^3\beta)=\\ O(-\eta_4^3\alpha,\eta_4^3\beta)=O(-i\alpha,-i\beta)=O(i\alpha,i\beta)=O(-\alpha,-\beta)=\\ O(\beta,\alpha)=O(\eta_4\beta,-\eta_4\alpha)=O(-\eta_4\beta,\eta_4\alpha)=O(\eta_4^3\beta,-\eta_4^3\alpha)=\\ O(-\eta_4^3\beta,\eta_4^3\alpha)=O(-i\beta,-i\alpha)=O(i\beta,i\alpha)=O(-\beta,-\alpha)\end{array}},$$

$$\langle\nabla_2+\nabla_3\rangle,\ \langle\nabla_2+\nabla_3+\nabla_4+\alpha\nabla_5\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_2+\nabla_3+\nabla_5\rangle,$$

$$\langle\nabla_2+\nabla_3+\alpha\nabla_5+\nabla_6\rangle^{O(\alpha)=O(\alpha^{-1})},$$

$$\langle\nabla_2+\nabla_4+\alpha\nabla_5+\beta\nabla_7+\nabla_8\rangle^{\begin{array}{l}O(\alpha,\beta)=O(\eta_4^3\alpha,-\eta_4^3\beta)=O(-\eta_4^3\alpha,\eta_4^3\beta)=O(\eta_4\alpha,-\eta_4\beta)=\\ O(-\eta_4\alpha,\eta_4\beta)=O(i\alpha,i\beta)=O(-i\alpha,-i\beta)=O(-\alpha,-\beta)\end{array}},$$

$$\langle\nabla_2+\nabla_5+\alpha\nabla_7+\nabla_8\rangle^{\begin{array}{l}O(\alpha)=O(\eta_5^2\alpha)=O(\eta_5^4\alpha)=\\ O(-\eta_5\alpha)=O(-\eta_5^3\alpha)\end{array}},\ \langle\nabla_3\rangle,$$

$$\langle\nabla_3+\nabla_4+\alpha\nabla_5\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},$$

$$\langle\nabla_3+\nabla_4+\alpha\nabla_6+\nabla_7\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_3+\nabla_5\rangle,\ \langle\nabla_3+\nabla_6\rangle,$$

$$\langle\nabla_3+\nabla_6+\nabla_7\rangle,\ \langle\nabla_4+\nabla_5+\alpha\nabla_7+\nabla_8\rangle^{O(\alpha)=O(-\alpha)},\ \langle\nabla_4+\nabla_6+\nabla_7\rangle,$$

$$\langle\nabla_4+\nabla_7\rangle,\ \langle\nabla_4+\nabla_7+\nabla_8\rangle,\ \langle\nabla_4+\nabla_8\rangle,\ \langle\nabla_5+\nabla_7+\nabla_8\rangle,\ \langle\nabla_5+\nabla_8\rangle,$$
$$\langle\nabla_6+\nabla_7\rangle,\ \langle\nabla_7\rangle,\ \langle\nabla_7+\nabla_8\rangle,\ \langle\nabla_8\rangle,$$

which gives the following new algebras:

$\mathbf{N}_{246}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_2e_2=e_5$	
		$e_2e_3=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{247}^{\alpha,\beta}$	:	$e_1e_1=e_5$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=e_5$	
		$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{248}$	:	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{249}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$		
		$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{250}$	:	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$	$e_2e_3=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{251}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$		
		$e_2e_3=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{252}^{\alpha,\beta}$	:	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	
		$e_3e_3=e_4$	$e_3e_4=\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{253}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_2e_3=e_5$		
		$e_3e_3=e_4$	$e_3e_4=\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{254}$	:	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{255}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4$
$\mathbf{N}_{256}^{\alpha}$	:	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=e_5$		
		$e_2e_4=\alpha e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{257}$	:	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_3=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{258}$	:	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{259}$	:	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{260}^{\alpha}$	:	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=e_5$		
		$e_3e_3=e_4$	$e_3e_4=\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{261}$	:	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{262}$	:	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{263}$	:	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{264}$	:	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_3e_3=e_4$	$e_4e_4=e_5$	
$\mathbf{N}_{265}$	:	$e_1e_2=e_3$	$e_2e_3=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{266}$	:	$e_1e_2=e_3$	$e_2e_3=e_5$	$e_3e_3=e_4$	$e_4e_4=e_5$	
$\mathbf{N}_{267}$	:	$e_1e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{268}$	:	$e_1e_2=e_3$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{269}$	:	$e_1e_2=e_3$	$e_3e_3=e_4$	$e_3e_4=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{270}$	:	$e_1e_2=e_3$	$e_3e_3=e_4$	$e_4e_4=e_5$		

3.13. **1-dimensional central extensions of $\mathbf{N}_{18}^{4*}$.** Here we will collect all information about $\mathbf{N}_{18}^{4*}$:

$\mathbf{N}_{18}^{4*}$	$e_1e_1=e_4$ $e_1e_2=e_3$ $e_3e_3=e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{18}^{4*}) = \langle[\Delta_{11}],[\Delta_{13}],[\Delta_{22}],[\Delta_{23}]\rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{18}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{18}^{4*})\oplus \langle[\Delta_{14}],[\Delta_{24}],[\Delta_{34}],[\Delta_{44}]\rangle$	$\phi_{\pm}=\begin{pmatrix} x & 0 & 0 & 0\\ 0 & \pm1 & 0 & 0\\ 0 & 0 & \pm x & 0\\ t & s & 0 & x^2\end{pmatrix}$

Let us use the following notations:

$$\nabla_1=[\Delta_{11}],\quad \nabla_2=[\Delta_{13}],\quad \nabla_3=[\Delta_{14}],\quad \nabla_4=[\Delta_{22}],$$
$$\nabla_5=[\Delta_{23}],\quad \nabla_6=[\Delta_{24}],\quad \nabla_7=[\Delta_{34}],\quad \nabla_8=[\Delta_{44}].$$

Take $\theta = \sum_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^{4*}_{18})$. Since

$$\phi_{\pm}^T \begin{pmatrix} \alpha_1 & 0 & \alpha_2 & \alpha_3 \\ 0 & \alpha_4 & \alpha_5 & \alpha_6 \\ \alpha_2 & \alpha_5 & 0 & \alpha_7 \\ \alpha_3 & \alpha_6 & \alpha_7 & \alpha_8 \end{pmatrix} \phi_{\pm} = \begin{pmatrix} \alpha_1^* & \alpha* & \alpha_2^* & \alpha_3^* \\ \alpha* & \alpha_4^* & \alpha_5^* & \alpha_6^* \\ \alpha_2^* & \alpha_5^* & 0 & \alpha_7^* \\ \alpha_3^* & \alpha_6^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

we have

$$\begin{array}{ll} \alpha_1^* = \alpha_1 x^2 + 2\alpha_3 xt + \alpha_8 t^2, & \alpha_2^* = \pm(\alpha_2 x + \alpha_7 t)x, \\ \alpha_3^* = (\alpha_3 x + \alpha_8 t)x^2, & \alpha_4^* = \alpha_4 \pm 2\alpha_6 s + \alpha_8 s^2, \\ \alpha_5^* = (\alpha_5 \pm \alpha_7 s)x, & \alpha_6^* = (\pm\alpha_6 + \alpha_8 s)x^2, \\ \alpha_7^* = \pm\alpha_7 x^3, & \alpha_8^* = \alpha_8 x^4. \end{array}$$

We are interested in $(\alpha_3, \alpha_6, \alpha_7, \alpha_8) \neq (0,0,0,0)$. Let us consider $\phi = \phi_+$ and the following cases:

1. $\alpha_8 = 0, \alpha_7 = 0, \alpha_6 = 0$, then $\alpha_3 \neq 0$ and choosing $t = -\frac{\alpha_1}{2\alpha_3}x$, we get $\alpha_1^* = 0$. Now consider the following subcases:
   (a) if $\alpha_2 = 0, \alpha_4 = 0, \alpha_5 = 0$, then we have the representative $\langle \nabla_3 \rangle$;
   (b) if $\alpha_2 = 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_5}{\alpha_3}}, s = 0, t = -\frac{\alpha_1\sqrt{\alpha_5}}{2\alpha_3\sqrt{\alpha_3}}$, we have the representative $\langle \nabla_3 + \nabla_5 \rangle$;
   (c) if $\alpha_2 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[3]{\frac{\alpha_4}{\alpha_3}}, s = 0, t = -\frac{\alpha_1\sqrt[3]{\alpha_4}}{2\alpha_3\sqrt[3]{\alpha_3}}$, we have the family of representatives $\langle \nabla_3 + \nabla_4 + \alpha\nabla_5 \rangle$;
   (d) if $\alpha_2 \neq 0$, then by choosing $x = \alpha_2\alpha_3^{-1}, s = 0, t = -\frac{\alpha_1\alpha_2}{2\alpha_3^2}$, we have the family of representatives $\langle \nabla_2 + \nabla_3 + \alpha\nabla_4 + \beta\nabla_5 \rangle$.
2. $\alpha_8 = 0, \alpha_7 = 0, \alpha_6 \neq 0$, then by choosing $s = -\frac{\alpha_4}{2\alpha_6}x$, we have $\alpha_4^* = 0$. Consider the following cases:
   (a) $\alpha_3 = 0$, then we have two families of representatives $\langle \alpha\nabla_1 + \beta\nabla_2 + \nabla_6 \rangle$ and $\langle \alpha\nabla_1 + \beta\nabla_2 + \nabla_5 + \nabla_6 \rangle$ depending on whether $\alpha_5 = 0$ or not;
   (b) $\alpha_3 \neq 0$ then by choosing $x = \frac{\alpha_6}{\alpha_3}, s = -\frac{\alpha_4}{2\alpha_6}, t = -\frac{\alpha_1\alpha_6}{2\alpha_3^2}$, we have the family of representatives $\langle \alpha\nabla_2 + \nabla_3 + \beta\nabla_5 + \nabla_6 \rangle$.
3. $\alpha_8 = 0, \alpha_7 \neq 0$, then by choosing $t = -\alpha_2\alpha_7^{-1}x, s = -\alpha_5\alpha_7^{-1}$, we have $\alpha_2^* = 0, \alpha_5^* = 0$. Thus, we can suppose that $\alpha_2 = 0, \alpha_5 = 0$ and now consider the following cases:
   (a) if $\alpha_1 = 0, \alpha_4 = 0, \alpha_6 = 0$, then we have the family of representatives $\langle \alpha\nabla_3 + \nabla_7 \rangle$;
   (b) if $\alpha_1 = 0, \alpha_4 = 0, \alpha_6 \neq 0$, then by choosing $x = \alpha_6\alpha_7^{-1}, s = 0, t = 0$, we have the family of representatives $\langle \alpha\nabla_3 + \nabla_6 + \nabla_7 \rangle$;
   (c) if $\alpha_1 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_4\alpha_7^{-1}}, s = 0, t = 0$, we have the family of representatives $\langle \alpha\nabla_3 + \nabla_4 + \beta\nabla_6 + \nabla_7 \rangle$;
   (d) if $\alpha_1 \neq 0$, then by choosing $x = \alpha_1\alpha_7^{-1}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \gamma\nabla_6 + \nabla_7 \rangle$.
4. $\alpha_8 \neq 0$, then by choosing $t = -\frac{\alpha_3}{\alpha_8}x, s = -\frac{\alpha_6}{\alpha_8}$, we have $\alpha_3^* = 0, \alpha_6^* = 0$. Thus, we can suppose that $\alpha_3 = 0, \alpha_6 = 0$. Consider the following cases:
   (a) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_5 = 0$, then we have the representatives $\langle \nabla_8 \rangle$ and $\langle \nabla_7 + \nabla_8 \rangle$ depending on whether $\alpha_7 = 0$ or not;
   (b) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_5\alpha_8^{-1}}, s = 0, t = 0$, we have the family of representatives $\langle \nabla_5 + \alpha\nabla_7 + \nabla_8 \rangle$;

(c) if $\alpha_1=0, \alpha_2=0, \alpha_4\neq 0$ then by choosing $x=\sqrt[4]{\alpha_4\alpha_8^{-1}}, s=0, t=0$, we have the family of representatives $\langle \nabla_4+\alpha\nabla_5+\beta\nabla_7+\nabla_8\rangle$;

(d) if $\alpha_1=0, \alpha_2\neq 0$, then by choosing $x=\sqrt{\alpha_2\alpha_8^{-1}}, s=0, t=0$, we have the family of representatives $\langle \nabla_2+\alpha\nabla_4+\beta\nabla_5+\gamma\nabla_7+\nabla_8\rangle$;

(e) if $\alpha_1\neq 0$, then by choosing $x=\sqrt{\alpha_1{\alpha_8}^{-1}}, s=0, t=0$, we have the family of representatives $\langle \nabla_1+\alpha\nabla_2+\beta\nabla_4+\gamma\nabla_5+\mu\nabla_7+\nabla_8\rangle$.

Summarizing all cases, we have the following distinct orbits:

$$\langle \nabla_1+\alpha\nabla_2+\beta\nabla_4+\gamma\nabla_5+\mu\nabla_7+\nabla_8\rangle^{\substack{O(\alpha,\beta,\gamma,\mu)=O(-\alpha,-\beta,-\gamma,\mu)=\\ O(-\alpha,\beta,\gamma,-\mu)=O(\alpha,-\beta,-\gamma,-\mu)}},$$
$$\langle \alpha\nabla_1+\beta\nabla_2+\nabla_5+\nabla_6\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)}, \langle \alpha\nabla_1+\beta\nabla_2+\nabla_6\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)},$$
$$\langle \nabla_1+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_6+\nabla_7\rangle^{O(\alpha,\beta,\gamma)=O(-\alpha,\beta,-\gamma)},$$
$$\langle \nabla_2+\nabla_3+\alpha\nabla_4+\beta\nabla_5\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)}, \langle \alpha\nabla_2+\nabla_3+\beta\nabla_5+\nabla_6\rangle,$$
$$\langle \nabla_2+\alpha\nabla_4+\beta\nabla_5+\gamma\nabla_7+\nabla_8\rangle^{O(\alpha,\beta,\gamma)=O(\alpha,i\beta,i\gamma)=O(\alpha,-i\beta,-i\gamma)}, \langle \nabla_3\rangle,$$
$$\langle \nabla_3+\nabla_4+\alpha\nabla_5\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},$$
$$\langle \alpha\nabla_3+\nabla_4+\beta\nabla_6+\nabla_7\rangle^{\substack{O(\alpha,\beta)=O(-\alpha,-\beta)=O(-\alpha,\eta_3\beta)=\\ O(-\alpha,-\eta_3^2\beta)=O(\alpha,-\eta_3\beta)=O(\alpha,\eta_3^2\beta)}}, \langle \nabla_3+\nabla_5\rangle,$$
$$\langle \alpha\nabla_3+\nabla_6+\nabla_7\rangle^{O(\alpha,\beta)=O(-\alpha,\beta)}, \langle \alpha\nabla_3+\nabla_7\rangle^{O(\alpha)=O(-\alpha)},$$
$$\langle \nabla_4+\alpha\nabla_5+\beta\nabla_7+\nabla_8\rangle^{\substack{O(\alpha,\beta)=O(i\alpha,-i\beta)=O(-i\alpha,i\beta)=O(-\alpha,-\beta)=\\ O(\alpha,-\beta)=O(i\alpha,i\beta)=O(-i\alpha,-i\beta)=O(-\alpha,\beta)}},$$
$$\langle \nabla_5+\alpha\nabla_7+\nabla_8\rangle^{O(\alpha)=O(\eta_3\alpha)=O(-\eta_3^2\alpha)=O(-\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)}, \langle \nabla_7+\nabla_8\rangle, \langle \nabla_8\rangle,$$

which gives the following new algebras:

$\mathbf{N}_{271}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_4+e_5$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_2e_2=\beta e_5$	
		$e_2e_3=\gamma e_5$	$e_3e_3=e_4$	$e_3e_4=\mu e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{272}^{\alpha,\beta}$	:	$e_1e_1=e_4+\alpha e_5$	$e_1e_2=e_3$	$e_1e_3=\beta e_5$		
		$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{273}^{\alpha,\beta}$	:	$e_1e_1=e_4+\alpha e_5$	$e_1e_2=e_3$	$e_1e_3=\beta e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{274}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_4+e_5$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=\beta e_5$	
		$e_2e_4=\gamma e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{275}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$	
		$e_2e_2=\alpha e_5$	$e_2e_3=\beta e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{276}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$	
		$e_2e_3=\beta e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{277}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_2e_2=\alpha e_5$	
		$e_2e_3=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{278}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{279}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=e_5$		
		$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{280}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=e_5$	
		$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{281}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_3=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{282}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$		
		$e_2e_4=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{283}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{284}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	
		$e_3e_3=e_4$	$e_3e_4=\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{285}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_2e_3=e_5$		

		$e_3e_3 = e_4$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{286}$	:	$e_1e_1 = e_4$	$e_1e_2 = e_3$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{287}$	:	$e_1e_1 = e_4$	$e_1e_2 = e_3$	$e_3e_3 = e_4$	$e_4e_4 = e_5$	

3.14. **1-dimensional central extensions of $\mathbf{N}_{19}^{4*}$.** Here we will collect all information about $\mathbf{N}_{19}^{4*}$:

$\mathbf{N}_{19}^{4*}$	$e_1e_1 = e_4 \quad e_1e_2 = e_3$ $e_2e_2 = e_4 \quad e_3e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{19}^{4*}) = \langle [\Delta_{11}], [\Delta_{13}], [\Delta_{22}], [\Delta_{23}] \rangle$ $\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{19}^{4*}) = \mathrm{H}^2_{\mathfrak{D}}(\mathbf{N}_{19}^{4*}) \oplus \langle [\Delta_{14}], [\Delta_{24}], [\Delta_{34}], [\Delta_{44}] \rangle$
$\phi_1 = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & q & 0 & 0 \\ 0 & 0 & xq & 0 \\ t & s & 0 & 1 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0 & p & 0 & 0 \\ y & 0 & 0 & 0 \\ 0 & 0 & yp & 0 \\ t & s & 0 & 1 \end{pmatrix},$ $x^2 = 1, q^2 = 1 \qquad y^2 = 1, p^2 = 1$		

Let us use the following notations:

$$\nabla_1 = [\Delta_{11}], \quad \nabla_2 = [\Delta_{13}], \quad \nabla_3 = [\Delta_{14}], \quad \nabla_4 = [\Delta_{22}],$$
$$\nabla_5 = [\Delta_{23}], \quad \nabla_6 = [\Delta_{24}], \quad \nabla_7 = [\Delta_{34}], \quad \nabla_8 = [\Delta_{44}].$$

Take $\theta = \sum_{i=1}^{8} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{19}^{4*})$. Since

$$\phi^T \begin{pmatrix} \alpha_1 & 0 & \alpha_2 & \alpha_3 \\ 0 & \alpha_4 & \alpha_5 & \alpha_6 \\ \alpha_2 & \alpha_5 & 0 & \alpha_7 \\ \alpha_3 & \alpha_6 & \alpha_7 & \alpha_8 \end{pmatrix} \phi = \begin{pmatrix} \alpha_1^* & \alpha^* & \alpha_2^* & \alpha_3^* \\ \alpha^* & \alpha_4^* & \alpha_5^* & \alpha_6^* \\ \alpha_2^* & \alpha_5^* & 0 & \alpha_7^* \\ \alpha_3^* & \alpha_6^* & \alpha_7^* & \alpha_8^* \end{pmatrix},$$

then, in the case $\phi = \phi_1^{x=1,q=1}$, we have

$$\alpha_1^* = \alpha_1 + 2\alpha_3 t + \alpha_8 t^2, \quad \alpha_2^* = \alpha_2 + \alpha_7 t, \quad \alpha_3^* = \alpha_3 + \alpha_8 t, \quad \alpha_4^* = \alpha_4 + 2\alpha_6 s + \alpha_8 s^2,$$
$$\alpha_5^* = \alpha_5 + \alpha_7 s, \quad \alpha_6^* = \alpha_6 + \alpha_8 s, \quad \alpha_7^* = \alpha_7, \quad \alpha_8^* = \alpha_8.$$

For define the main families of representatives, we will use $\phi = \phi_1^{x=1,q=1}$ and for find equal orbits we will use other automorphisms. We are interested in

$$(\alpha_3, \alpha_6, \alpha_7, \alpha_8) \neq (0, 0, 0, 0).$$

Let us consider the following cases:

1. if $\alpha_8 = 0, \alpha_7 = 0, \alpha_6 = 0$, then $\alpha_3 \neq 0$ and choosing $t = -\frac{\alpha_1}{2\alpha_3}$, we have the family of representatives $\langle \alpha\nabla_2 + \nabla_3 + \beta\nabla_4 + \gamma\nabla_5 \rangle$;
2. if $\alpha_8 = 0, \alpha_7 = 0, \alpha_6 \neq 0$ and $\alpha_3 = 0$, then by choosing some suitable automorphism ϕ_2 we have $\alpha_3^* \neq 0$, $\alpha_6^* = 0$, which is the case considered above;
3. if $\alpha_8 = 0, \alpha_7 = 0, \alpha_6 \neq 0, \alpha_3 \neq 0$, then by choosing $t = -\frac{\alpha_1}{2\alpha_3}, s = \frac{\alpha_4}{2\alpha_6}$, we have the family of representatives $\langle \alpha\nabla_2 + \beta\nabla_3 + \gamma\nabla_5 + \nabla_6 \rangle_{\beta \neq 0}$;
4. if $\alpha_8 = 0, \alpha_7 \neq 0$, then by choosing $t = -\alpha_2 {\alpha_7}^{-1}, s = -\alpha_5 {\alpha_7}^{-1}$, we have the family of representatives $\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_6 + \nabla_7 \rangle$;
5. if $\alpha_8 \neq 0$, then by choosing $t = -\alpha_3 {\alpha_8}^{-1}, s = -\alpha_6 {\alpha_8}^{-1}$, we have the family of representatives $\langle \alpha\nabla_1 + \beta\nabla_2 + \gamma\nabla_4 + \mu\nabla_5 + \nu\nabla_7 + \nabla_8 \rangle$.

Summarizing, we have the following distinct orbits:

$$\langle \alpha\nabla_1+\beta\nabla_2+\gamma\nabla_4+\mu\nabla_5+\nu\nabla_7+\nabla_8\rangle^{\substack{O(\alpha,\beta,\gamma,\mu,\nu)=O(\alpha,-\beta,\gamma,\mu,-\nu)=\\ O(\alpha,\beta,\gamma,-\mu,-\nu)=O(\alpha,-\beta,\gamma,-\mu,\nu)=\\ O(\gamma,\mu,\alpha,\beta,\nu)=O(\gamma,-\mu,\alpha,\beta,-\nu)=\\ O(\gamma,\mu,\alpha,-\beta,-\nu)=O(\gamma,-\mu,\alpha,-\beta,\nu)}},$$

$$\langle \alpha\nabla_1+\beta\nabla_3+\gamma\nabla_4+\mu\nabla_6+\nabla_7\rangle^{\substack{O(\alpha,\beta,\gamma,\mu)=O(-\alpha,-\beta,-\gamma,\mu)=\\ O(-\alpha,\beta,-\gamma,-\mu)=O(\alpha,-\beta,\gamma,-\mu)=\\ O(\gamma,\mu,\alpha,\beta)=O(-\gamma,-\mu,-\alpha,\beta)=\\ O(-\gamma,\mu,-\alpha,-\beta)=O(\gamma,-\mu,\alpha,-\beta)}},$$

$$\langle \alpha\nabla_2+\nabla_3+\beta\nabla_4+\gamma\nabla_5\rangle^{O(\alpha,\beta,\gamma)=O(-\alpha,\beta,\gamma)=O(-\alpha,-\beta,\gamma)=O(\alpha,-\beta,\gamma)=},$$

$$\langle \alpha\nabla_2+\beta\nabla_3+\gamma\nabla_5+\nabla_6\rangle_{\beta\neq 0}^{O(\alpha,\beta,\gamma)=O(\alpha,-\beta,-\gamma)=O(\frac{\gamma}{\beta},\frac{1}{\beta},\frac{\alpha}{\beta})=O(\frac{\gamma}{\beta},-\frac{1}{\beta},-\frac{\alpha}{\beta})},$$

which gives the following new algebras:

$\mathbf{N}_{288}^{\alpha,\beta,\gamma,\mu,\nu}$	:	$e_1e_1=e_4+\alpha e_5$	$e_1e_2=e_3$	$e_1e_3=\beta e_5$	$e_2e_2=e_4+\gamma e_5$
		$e_2e_3=\mu e_5$	$e_3e_3=e_4$	$e_3e_4=\nu e_5$	$e_4e_4=e_5$
$\mathbf{N}_{289}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_4+\alpha e_5$	$e_1e_2=e_3$	$e_1e_4=\beta e_5$	$e_2e_2=e_4+\gamma e_5$
		$e_2e_4=\mu e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{290}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$
		$e_2e_2=e_4+\beta e_5$	$e_2e_3=\gamma e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{291}^{\alpha,\beta\neq 0,\gamma}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=\beta e_5$
		$e_2e_3=\gamma e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	

4. Central extensions of nilpotent non-$\mathfrak{CCD}$-algebras.

4.1. 1-dimensional central extensions of $\mathbf{N}_{01}^4$.

Here we will collect all information about $\mathbf{N}_{01}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{01}^4$	$e_1e_1=e_2$ $e_1e_2=e_3$ $e_2e_3=e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{01}^4)=\Big\langle[\Delta_{ij}]\Big\rangle$ $(i,j)\notin\{(1,1),(1,2),(2,3)\}$	$\phi=\begin{pmatrix} x & 0 & 0 & 0\\ 0 & x^2 & 0 & 0\\ z & 0 & x^3 & 0\\ t & 0 & x^2z & x^5\end{pmatrix}$

Let us use the following notations:

$$\nabla_1=[\Delta_{13}],\quad \nabla_2=[\Delta_{14}],\quad \nabla_3=[\Delta_{22}],\quad \nabla_4=[\Delta_{24}],$$
$$\nabla_5=[\Delta_{33}],\quad \nabla_6=[\Delta_{34}],\quad \nabla_7=[\Delta_{44}].$$

Take $\theta=\sum\limits_{i=1}^{7}\alpha_i\nabla_i\in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{01}^4)$. Since

$$\phi^T\begin{pmatrix} 0 & 0 & \alpha_1 & \alpha_2\\ 0 & \alpha_3 & 0 & \alpha_4\\ \alpha_1 & 0 & \alpha_5 & \alpha_6\\ \alpha_2 & \alpha_4 & \alpha_6 & \alpha_7\end{pmatrix}\phi=\begin{pmatrix} \alpha^* & \alpha^{**} & \alpha_1^* & \alpha_2^*\\ \alpha^{**} & \alpha_3^* & \alpha^{***} & \alpha_4^*\\ \alpha_1^* & \alpha^{***} & \alpha_5^* & \alpha_6^*\\ \alpha_2^* & \alpha_4^* & \alpha_6^* & \alpha_7^*\end{pmatrix}$$

we have

$$\begin{array}{lll}
\alpha_1^*=\big((\alpha_1x+\alpha_5z+\alpha_6t)x+(\alpha_2x+\alpha_6z+\alpha_7t)z\big)x^2, & & \\
\alpha_2^*=(\alpha_2x+\alpha_6z+\alpha_7t)x^5, & \alpha_3^*=\alpha_3x^4, & \alpha_4^*=\alpha_4x^7,\\
\alpha_5^*=(\alpha_5x^2+2\alpha_6xz+\alpha_7z^2)x^4, & \alpha_6^*=(\alpha_6x+\alpha_7z)x^7, & \alpha_7^*=\alpha_7x^{10}.
\end{array}$$

We are interested in $(\alpha_2,\alpha_4,\alpha_6,\alpha_7)\neq(0,0,0,0)$ and consider following cases:

1. $\alpha_7=\alpha_6=\alpha_4=0$, then $\alpha_2\neq 0$, and we have the following subcases:
   (a) if $\alpha_5=-\alpha_2$, then we have
      (i) if $\alpha_1=0,\alpha_3=0$, then we have the representative $\langle\nabla_2-\nabla_5\rangle$;

(ii) if $\alpha_1 = 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt{\alpha_3 \alpha_2{}^{-1}}, z = 0, t = 0$, we have the representative $\langle \nabla_2 + \nabla_3 - \nabla_5 \rangle$;

(iii) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt{\alpha_1 \alpha_2{}^{-1}}, z = 0, t = 0$, we have the family of representatives $\langle \nabla_1 + \nabla_2 + \alpha \nabla_3 - \nabla_5 \rangle$;

(b) if $\alpha_5 \neq -\alpha_2$, then by choosing $z = -\frac{\alpha_1}{\alpha_5+\alpha_2} x, t = 0$, we have the family of representatives $\langle \nabla_2 + \alpha \nabla_5 \rangle_{\alpha \neq -1}$ and $\langle \nabla_2 + \nabla_3 + \alpha \nabla_5 \rangle_{\alpha \neq -1}$ depending on whether $\alpha_3 = 0$ or not, which will be jointed with the cases (1(a)i) and (1(a)ii).

2. $\alpha_7 = 0, \alpha_6 = 0, \alpha_4 \neq 0$, then we have the following subcases:
   (a) if $\alpha_5 = -\alpha_2, \alpha_1 = 0$,
      (i) if $\alpha_3 = 0$, then we have the representatives $\langle \nabla_4 \rangle$ and $\langle \nabla_2 + \nabla_4 - \nabla_5 \rangle$ depending on whether $\alpha_2 = 0$ or not;
      (ii) if $\alpha_3 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_3 \alpha_4{}^{-1}}$, we have the family of representatives $\langle \alpha \nabla_2 + \nabla_3 + \nabla_4 - \alpha \nabla_5 \rangle$;
   (b) if $\alpha_5 = -\alpha_2, \alpha_1 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_1 \alpha_4{}^{-1}}$, we have the family of representatives $\langle \nabla_1 + \alpha \nabla_2 + \beta \nabla_3 + \nabla_4 - \alpha \nabla_5 \rangle$;
   (c) if $\alpha_5 \neq -\alpha_2$, then we have
      (i) if $\alpha_3 = 0, \alpha_2 = 0$, then by choosing $x = \frac{\alpha_5}{\alpha_4}, z = -\frac{\alpha_1 \alpha_5}{\alpha_4(\alpha_2+\alpha_5)}, t = 0$, we have the representative $\langle \nabla_4 + \nabla_5 \rangle$;
      (ii) if $\alpha_3 = 0, \alpha_2 \neq 0$, then by choosing $x = \frac{\alpha_2}{\alpha_4}, z = -\frac{\alpha_1 \alpha_2}{\alpha_4(\alpha_2+\alpha_5)}, t = 0$, we have the family of representatives $\langle \nabla_2 + \nabla_4 + \alpha \nabla_5 \rangle_{\alpha \neq -1}$, which will be jointed with a representative from the case (2(a)i);
      (iii) if $\alpha_3 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_3 \alpha_4{}^{-1}}, z = -\frac{\alpha_1 \sqrt[3]{\alpha_3}}{(\alpha_2+\alpha_5)\sqrt[3]{\alpha_4}}, t = 0$, we have the family of representatives $\langle \alpha \nabla_2 + \nabla_3 + \nabla_4 + \beta \nabla_5 \rangle_{\beta \neq -\alpha}$, which will be jointed with the family from the case (2(a)i).
3. $\alpha_7 = 0, \alpha_6 \neq 0$, then we consider the following subcases:
   (a) if $\alpha_3 = 0, \alpha_4 = 0$, then choosing $z = -\frac{\alpha_2}{\alpha_6} x, t = -\frac{\alpha_1 x + \alpha_5 z}{\alpha_6}$, we have representatives $\langle \nabla_6 \rangle$ and $\langle \nabla_5 + \nabla_6 \rangle$ depending on whether $\alpha_5 = 2\alpha_2$ or not;
   (b) if $\alpha_3 = 0, \alpha_4 \neq 0$, then by choosing $x = \frac{\alpha_4}{\alpha_6}, z = -\frac{\alpha_2 \alpha_4}{\alpha_6^2}, t = \frac{\alpha_4(\alpha_2\alpha_5 - \alpha_1\alpha_6)}{\alpha_6^3}$, we have the family of representatives $\langle \nabla_4 + \alpha \nabla_5 + \nabla_6 \rangle$;
   (c) if $\alpha_3 \neq 0$, then by choosing $x = \sqrt[4]{\frac{\alpha_3}{\alpha_6}}, z = -\frac{\alpha_2 \sqrt[4]{\alpha_3}}{\alpha_6 \sqrt[4]{\alpha_6}}, t = \frac{(\alpha_2\alpha_5 - \alpha_1\alpha_6)\sqrt[4]{\alpha_3}}{\alpha_6^2 \sqrt[4]{\alpha_6}}$, we have the family of representatives $\langle \nabla_3 + \alpha \nabla_4 + \beta \nabla_5 + \nabla_6 \rangle$.
4. $\alpha_7 \neq 0$, then by choosing $z = -\frac{\alpha_6}{\alpha_7} x, t = \frac{\alpha_6^2 + \alpha_2 \alpha_7}{\alpha_7^2} x$, we have $\alpha_2^* = 0, \alpha_6^* = 0$. Thus, we can suppose that $\alpha_2 = 0, \alpha_6 = 0$ and now consider following subcases:
   (a) $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0$, then we have representatives $\langle \nabla_7 \rangle$ and $\langle \nabla_5 + \nabla_7 \rangle$ depending on whether $\alpha_5 \alpha_7 - \alpha_6^2 = 0$ or not;
   (b) $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_4 \alpha_7{}^{-1}}$, we have the family of representatives $\langle \nabla_4 + \alpha \nabla_5 + \nabla_7 \rangle$;
   (c) $\alpha_1 = 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt[6]{\alpha_3 \alpha_7{}^{-1}}$, we have the family of representatives $\langle \nabla_3 + \alpha \nabla_4 + \beta \nabla_5 + \nabla_7 \rangle$;
   (d) $\alpha_1 \neq 0$, then by choosing $x = \sqrt[6]{\alpha_1 \alpha_7^{-3}}$, we have the family of representatives $\langle \nabla_1 + \alpha \nabla_3 + \beta \nabla_4 + \gamma \nabla_5 + \nabla_7 \rangle$.

Summarizing all cases, we have the following distinct orbits:

$$\langle \nabla_1+\alpha\nabla_2+\beta\nabla_3+\nabla_4-\alpha\nabla_5\rangle^{O(\alpha,\beta)=O(-\eta_3\alpha,\eta_3\beta)=O(\eta_3^2\alpha,-\eta_3^2\beta)},$$
$$\langle \nabla_1+\nabla_2+\alpha\nabla_3-\nabla_5\rangle,$$
$$\langle \nabla_1+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\nabla_7\rangle^{\substack{O(\alpha,\beta,\gamma)=O(\alpha,\beta,-\eta_3\gamma)=O(\alpha,-\beta,-\eta_3\gamma)=\\ O(\alpha,-\beta,\eta_3^2\gamma)=O(\alpha,\beta,\eta_3^2\gamma)=O(\alpha,-\beta,\gamma)}},$$
$$\langle \alpha\nabla_2+\nabla_3+\nabla_4+\beta\nabla_5\rangle^{O(\alpha,\beta)=O(-\eta_3\alpha,-\eta_3\beta)=O(\eta_3^2\alpha,\eta_3^2\beta)},\ \langle \nabla_2+\nabla_3+\alpha\nabla_5\rangle,$$
$$\langle \nabla_2+\nabla_4+\alpha\nabla_5\rangle,\ \langle \nabla_2+\alpha\nabla_5\rangle,\ \langle \nabla_3+\alpha\nabla_4+\beta\nabla_5+\nabla_6\rangle^{\substack{O(\alpha,\beta)=O(-i\alpha,-\beta)=\\ O(i\alpha,-\beta)=O(-\alpha,\beta)}},$$
$$\langle \nabla_3+\alpha\nabla_4+\beta\nabla_5+\nabla_7\rangle^{\substack{O(\alpha,\beta)=O(\alpha,-\eta_3\beta)=O(-\alpha,-\eta_3\beta)=\\ O(-\alpha,\eta_3^2\beta)=O(\alpha,\eta_3^2\beta)=O(-\alpha,\beta)}},\ \langle\nabla_4\rangle,\ \langle \nabla_4+\nabla_5\rangle,$$
$$\langle \nabla_4+\alpha\nabla_5+\nabla_6\rangle,\ \langle \nabla_4+\alpha\nabla_5+\nabla_7\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle \nabla_5+\nabla_6\rangle,\ \langle \nabla_5+\nabla_7\rangle,$$
$$\langle\nabla_6\rangle,\ \langle\nabla_7\rangle.$$

Hence, we have the following new algebras:

$\mathbf{N}_{292}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=\alpha e_5$	
		$e_2e_2=\beta e_5$	$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=-\alpha e_5$	
$\mathbf{N}_{293}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$	
		$e_2e_2=\alpha e_5$	$e_2e_3=e_4$	$e_3e_3=-e_5$		
$\mathbf{N}_{294}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_2e_2=\alpha e_5$	
		$e_2e_3=e_4$	$e_2e_4=\beta e_5$	$e_3e_3=\gamma e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{295}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=e_5$	
		$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\beta e_5$		
$\mathbf{N}_{296}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$		
		$e_2e_2=e_5$	$e_2e_3=e_4$	$e_3e_3=\alpha e_5$		
$\mathbf{N}_{297}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$		
		$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$		
$\mathbf{N}_{298}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_3=e_4$	$e_3e_3=\alpha e_5$
$\mathbf{N}_{299}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=e_4$	
		$e_2e_4=\alpha e_4$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{300}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=e_4$	
		$e_2e_4=\alpha e_4$	$e_3e_3=\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{301}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_2e_4=e_5$	
$\mathbf{N}_{302}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{303}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$		
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{304}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$		
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{305}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=e_5$	$e_3e_4=e_5$
$\mathbf{N}_{306}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{307}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_4e_4=e_5$	
$\mathbf{N}_{308}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_4e_4=e_5$	

4.2. **1-dimensional central extensions of $\mathbf{N}_{02}^4$.** Here we will collect all information about $\mathbf{N}_{02}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{02}^4$	$e_1e_1=e_2$ $e_1e_2=e_3$ $e_1e_3=e_4$ $e_2e_3=e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{02}^4)=\Big\langle[\Delta_{ij}]\Big\rangle$ $(i,j)\notin\{(1,1),(1,2),(1,3)\}$	$\phi=\begin{pmatrix}1&0&0&0\\0&1&0&0\\z&0&1&0\\t&2z&z&1\end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{14}], & \nabla_2 = [\Delta_{22}], & \nabla_3 = [\Delta_{23}], & \nabla_4 = [\Delta_{24}], \\ \nabla_5 = [\Delta_{33}], & \nabla_6 = [\Delta_{34}], & \nabla_7 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{7} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^4_{02})$. Since

$$\phi^T \begin{pmatrix} 0 & 0 & 0 & \alpha_1 \\ 0 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & \alpha_3 & \alpha_5 & \alpha_6 \\ \alpha_1 & \alpha_4 & \alpha_6 & \alpha_7 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha^{**} & \alpha^{***} & \alpha_1^* \\ \alpha^{**} & \alpha_2^* & \alpha_3^* + \alpha^{***} & \alpha_4^* \\ \alpha^{***} & \alpha_3^* + \alpha^{***} & \alpha_5^* & \alpha_6^* \\ \alpha_1^* & \alpha_4^* & \alpha_6^* & \alpha_7^* \end{pmatrix}$$

we have

$$\begin{array}{lll} \alpha_1^* & = & \alpha_1 + \alpha_6 z + \alpha_7 t, \\ \alpha_2^* & = & \alpha_2 + 4\alpha_4 z + 4\alpha_7 z^2, \\ \alpha_3^* & = & \alpha_3 + 2\alpha_6 z + (\alpha_4 + 2\alpha_7 z)z - (\alpha_5 z + \alpha_6 t) - (\alpha_1 + \alpha_6 z + \alpha_7 t)z, \\ \alpha_4^* & = & \alpha_4 + 2\alpha_7 z, \\ \alpha_5^* & = & \alpha_5 + 2\alpha_6 z + \alpha_7 z^2, \\ \alpha_6^* & = & \alpha_6 + \alpha_7 z, \\ \alpha_7^* & = & \alpha_7. \end{array}$$

We are interested in $(\alpha_1, \alpha_4, \alpha_6, \alpha_7) \neq (0,0,0,0)$ and consider following cases:

1. if $\alpha_7 = \alpha_6 = \alpha_4 = 0$, then $\alpha_1 \neq 0$, and we have
   (a) if $\alpha_5 = -\alpha_1$, then we have the family of representatives
   $$\langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_3 - \nabla_5 \rangle;$$
   (b) if $\alpha_5 \neq -\alpha_1$, then by choosing $z = -\frac{\alpha_3}{\alpha_1+\alpha_5}, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_5 \rangle_{\beta\neq -1}$;
2. if $\alpha_7 = 0, \alpha_6 = 0, \alpha_4 \neq 0$, then by choosing $z = -\frac{\alpha_2}{4\alpha_4}, t = 0$, we have the family of representatives $\langle \alpha\nabla_1 + \beta\nabla_3 + \nabla_4 + \gamma\nabla_5 \rangle$;
3. if $\alpha_7 = 0, \alpha_6 \neq 0$, then by choosing
   $$z = -\alpha_1 \alpha_6{}^{-1}, t = (\alpha_3\alpha_6 - \alpha_1(2\alpha_6 + \alpha_4 - \alpha_5))\alpha_6^{-1},$$
   we have the family of representatives $\langle \alpha\nabla_2 + \beta\nabla_4 + \gamma\nabla_5 + \nabla_6 \rangle$;
4. if $\alpha_7 \neq 0$, then by choosing $z = -\alpha_6\alpha_7{}^{-1}, t = (\alpha_6^2 - \alpha_1\alpha_7)\alpha_7^{-2}$, we have the family of representatives $\langle \alpha\nabla_2 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_7 \rangle$.

Summarizing, we have the following distinct orbits:

$$\langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_3 - \nabla_5 \rangle, \langle \nabla_1 + \alpha\nabla_2 + \beta\nabla_5 \rangle_{\beta\neq -1}, \langle \alpha\nabla_1 + \beta\nabla_3 + \nabla_4 + \gamma\nabla_5 \rangle,$$
$$\langle \alpha\nabla_2 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_7 \rangle, \langle \alpha\nabla_2 + \beta\nabla_4 + \gamma\nabla_5 + \nabla_6 \rangle,$$

which gives the following new algebras:

$\mathbf{N}_{309}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$
		$e_2e_2 = \alpha e_5$	$e_2e_3 = e_4 + \beta e_5$	$e_3e_3 = -e_5$	
$\mathbf{N}_{310}^{\alpha,\beta\neq-1}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$
		$e_2e_2 = \alpha e_5$	$e_2e_3 = e_4$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{311}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$
		$e_2e_3 = e_4 + \beta e_5$	$e_2e_4 = e_5$	$e_3e_3 = \gamma e_5$	
$\mathbf{N}_{312}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4 + \beta e_5$	$e_2e_4 = \gamma e_5$	$e_3e_3 = \mu e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{313}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \alpha e_5$

$$e_2e_3 = e_4 \qquad e_2e_4 = \beta e_5 \qquad e_3e_3 = \gamma e_5 \quad e_3e_4 = e_5$$

4.3. **1-dimensional central extensions of $\mathbf{N}_{03}^4$.** Here we will collect all information about $\mathbf{N}_{03}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{03}^4$	$e_1e_1 = e_2$ $e_1e_2 = e_3$ $e_3e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{03}^4) = \Big\langle [\Delta_{ij}] \Big\rangle$ $(i,j) \notin \{(1,1),(1,2),(3,3)\}$	$\phi = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & x^2 & 0 & 0 \\ 0 & 0 & x^3 & 0 \\ t & 0 & 0 & x^6 \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{13}], & \nabla_2 = [\Delta_{14}], & \nabla_3 = [\Delta_{22}], & \nabla_4 = [\Delta_{23}], \\ \nabla_5 = [\Delta_{24}], & \nabla_6 = [\Delta_{34}], & \nabla_7 = [\Delta_{44}]. & \end{array}$$

Take $\theta = \sum\limits_{i=1}^{7} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{03}^4)$. Since

$$\phi^T \begin{pmatrix} 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 & \alpha_5 \\ \alpha_1 & \alpha_4 & 0 & \alpha_6 \\ \alpha_2 & \alpha_5 & \alpha_6 & \alpha_7 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha^{**} & \alpha_1^* & \alpha_2^* \\ \alpha^{**} & \alpha_3^* & \alpha_4^* & \alpha_5^* \\ \alpha_1^* & \alpha_4^* & 0 & \alpha_6^* \\ \alpha_2^* & \alpha_5^* & \alpha_6^* & \alpha_7^* \end{pmatrix}$$

we have

$$\begin{array}{llll} \alpha_1^* = (\alpha_1 x + \alpha_6 t)x^3, & \alpha_2^* = (\alpha_2 x + \alpha_7 t)x^6, & \alpha_3^* = \alpha_3 x^4, & \alpha_4^* = \alpha_4 x^5, \\ \alpha_5^* = \alpha_5 x^8, & \alpha_6^* = \alpha_6 x^9, & \alpha_7^* = \alpha_7 x^{12}. & \end{array}$$

We are interested in $(\alpha_2, \alpha_5, \alpha_6, \alpha_7) \neq (0,0,0,0)$ and consider following cases:

1. $\alpha_7 = \alpha_6 = \alpha_5 = 0$, then $\alpha_2 \neq 0$, and we have the following subcases:
   (a) if $\alpha_1 = 0, \alpha_3 = 0$, then we have the representatives $\langle \nabla_2 \rangle$ and $\langle \nabla_2 + \nabla_4 \rangle$ depending on whether $\alpha_4 = 0$ or not;
   (b) if $\alpha_1 = 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_3 \alpha_2{}^{-1}}, t = 0$, we have the family of representatives $\langle \nabla_2 + \nabla_3 + \alpha \nabla_4 \rangle$;
   (c) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_1 \alpha_2^{-1}}, t = 0$, we have the family of representatives $\langle \nabla_1 + \nabla_2 + \alpha \nabla_3 + \beta \nabla_4 \rangle$.
2. $\alpha_7 = 0, \alpha_6 = 0, \alpha_5 \neq 0$, then we have the following subcases:
   (a) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0$, then we have the representatives $\langle \nabla_5 \rangle$ and $\langle \nabla_4 + \nabla_5 \rangle$ depending on whether $\alpha_5 = 0$ or not;
   (b) if $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_3 \alpha_5^{-1}}, t = 0$, we have the family of representatives $\langle \nabla_3 + \alpha \nabla_4 + \nabla_5 \rangle$;
   (c) if $\alpha_1 = 0, \alpha_2 \neq 0$, then by choosing $x = \alpha_2 \alpha_5^{-1}, t = 0$, we have the family of representatives $\langle \nabla_2 + \alpha \nabla_3 + \beta \nabla_4 + \nabla_5 \rangle$;
   (d) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_1 \alpha_5^{-1}}, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha \nabla_2 + \beta \nabla_3 + \gamma \nabla_4 + \nabla_5 \rangle$.
3. $\alpha_7 = 0, \alpha_6 \neq 0$, then we have the following subcases:
   (a) if $\alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$, then we have representatives $\langle \nabla_6 \rangle$ and $\langle \nabla_5 + \nabla_6 \rangle$ depending on whether $\alpha_5 = 0$ or not;
   (b) if $\alpha_2 = 0, \alpha_3 = 0, \alpha_4 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_4 \alpha_6{}^{-1}}, t = -\alpha_1 \sqrt[4]{\alpha_4 \alpha_6^{-5}}$, we have the family of representatives $\langle \nabla_4 + \alpha \nabla_5 + \nabla_6 \rangle$;

(c) if $\alpha_2=0,\alpha_3\neq 0$, then by choosing $x=\sqrt[5]{\alpha_3\alpha_6^{-1}}, t=-\alpha_1\sqrt[5]{\alpha_3\alpha_6^{-6}}$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_4+\beta\nabla_5+\nabla_6\rangle$;

(d) if $\alpha_2\neq 0$, then by choosing $x=\alpha_2\alpha_6^{-1}, t=-\alpha_1\sqrt{\alpha_2\alpha_6^{-3}}$, we have the family of representatives $\langle\nabla_2+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\nabla_6\rangle$.

4. $\alpha_7\neq 0$, then we have the following subcases:

(a) $\alpha_1\alpha_7-\alpha_2\alpha_6=0,\alpha_3=0,\alpha_4=0,\alpha_5=0$, then we have representatives $\langle\nabla_7\rangle$ and $\langle\nabla_6+\nabla_7\rangle$ depending on whether $\alpha_6=0$ or not;

(b) $\alpha_1\alpha_7-\alpha_2\alpha_6=0,\alpha_3=0,\alpha_4=0,\alpha_5\neq 0$, then by choosing $x=\sqrt[4]{\alpha_5\alpha_7^{-1}}$, $t=-\alpha_2\sqrt[4]{\alpha_5\alpha_7^{-5}}$, we have the family of representatives $\langle\nabla_5+\alpha\nabla_6+\nabla_7\rangle$;

(c) $\alpha_1\alpha_7-\alpha_2\alpha_6=0,\alpha_3=0,\alpha_4\neq 0$, then by choosing $x=\sqrt[7]{\alpha_4\alpha_7^{-1}}$, $t=-\alpha_2\sqrt[7]{\alpha_4\alpha_7^{-8}}$, we have the family of representatives $\langle\nabla_4+\alpha\nabla_5+\beta\nabla_6+\nabla_7\rangle$;

(d) $\alpha_1\alpha_7-\alpha_2\alpha_6=0,\alpha_3\neq 0$, then by choosing $x=\sqrt[8]{\alpha_3\alpha_7^{-1}}$, $t=-\alpha_2\sqrt[8]{\alpha_3\alpha_7^{-9}}$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_4+\beta\nabla_5+\gamma\nabla_6+\nabla_7\rangle$;

(e) $\alpha_1\alpha_7-\alpha_2\alpha_6\neq 0$, then by choosing
$$x=\sqrt[8]{(\alpha_1\alpha_7-\alpha_2\alpha_6)\alpha_7^{-2}}, t=-\alpha_2\sqrt[8]{(\alpha_1\alpha_7-\alpha_2\alpha_6)\alpha_7^{-10}},$$
we have the family of representatives $\langle\nabla_1+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\mu\nabla_6+\nabla_7\rangle$.

Summarizing, we have the following distinct orbits:

$$\langle\nabla_1+\nabla_2+\alpha\nabla_3+\beta\nabla_4\rangle^{O(\alpha,\beta)=O(\alpha,-\eta_3\beta)=O(\alpha,\eta_3^2\beta)},$$
$$\langle\nabla_1+\alpha\nabla_2+\beta\nabla_3+\gamma\nabla_4+\nabla_5\rangle^{O(\alpha,\beta,\gamma)=O(-i\alpha,\beta,i\gamma)=O(i\alpha,\beta,-i\gamma)=O(-\alpha,\beta,-\gamma)},$$
$$\langle\nabla_1+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\mu\nabla_6+\nabla_7\rangle^{\substack{O(\alpha,\beta,\gamma,\mu)=O(\alpha,\eta_4^3\beta,-\gamma,-\eta_4^3\mu)=\\ O(\alpha,-\eta_4^3\beta,-\gamma,\eta_4^3\mu)=O(\alpha,\eta_4\beta,-\gamma,-\eta_4\mu)=\\ O(\alpha,-\eta_4\beta,-\gamma,\eta_4\mu)=O(\alpha,i\beta,\gamma,i\mu)=\\ O(\alpha,-i\beta,\gamma,-i\mu)=O(\alpha,-\beta,\gamma,-\mu)}},$$
$$\langle\nabla_2\rangle, \langle\nabla_2+\nabla_3+\alpha\nabla_4\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)}, \langle\nabla_2+\alpha\nabla_3+\beta\nabla_4+\nabla_5\rangle,$$
$$\langle\nabla_2+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\nabla_6\rangle^{O(\alpha,\beta,\gamma)=O(-\alpha,\beta,-\gamma)}, \langle\nabla_2+\nabla_4\rangle,$$
$$\langle\nabla_3+\alpha\nabla_4+\nabla_5\rangle^{\substack{O(\alpha)=O(-\alpha)=\\ O(i\alpha)=O(-i\alpha)}},$$
$$\langle\nabla_3+\alpha\nabla_4+\beta\nabla_5+\nabla_6\rangle^{\substack{O(\alpha,\beta)=O(\eta_5^4\alpha,-\eta_5\beta)=O(-\eta_5^3\alpha,\eta_5^2\beta)=\\ O(\eta_5^2\alpha,-\eta_5^3\beta)=O(-\eta_5\alpha,\eta_5^4\beta)}},$$
$$\langle\nabla_3+\alpha\nabla_4+\beta\nabla_5+\gamma\nabla_6+\nabla_7\rangle^{\substack{O(\alpha,\beta,\gamma)=O(\eta_4^3\alpha,-\beta,-\eta_4^3\gamma)=O(-\eta_4^3\alpha,-\beta,\eta_4^3\gamma)=\\ O(\eta_4\alpha,-\beta,-\eta_4\gamma)=O(-\eta_4\alpha,-\beta,\eta_4\gamma)=\\ O(i\alpha,\beta,i\gamma)=O(-i\alpha,\beta,-i\gamma)=O(-\alpha,\beta,-\gamma)}},$$
$$\langle\nabla_4+\nabla_5\rangle, \langle\nabla_4+\alpha\nabla_5+\nabla_6\rangle^{O(\alpha)=O(i\alpha)=O(-\alpha)=O(-i\alpha)},$$
$$\langle\nabla_4+\alpha\nabla_5+\beta\nabla_6+\nabla_7\rangle^{\substack{O(\alpha,\beta)=O(\eta_7^4\alpha,-\eta_7^3\beta)=O(-\eta_7\alpha,\eta_7^6\beta)=O(-\eta_7^5\alpha,\eta_7^2\beta)=\\ O(\eta_7^2\alpha,-\eta_7^5\beta)=O(\eta_7^6\alpha,-\eta_7\beta)=O(-\eta_7^3\alpha,\eta_7^4\beta)}}, \langle\nabla_5\rangle,$$
$$\langle\nabla_5+\nabla_6\rangle, \langle\nabla_5+\alpha\nabla_6+\nabla_7\rangle^{O(\alpha)=O(i\alpha)=O(-\alpha)=O(-i\alpha)}, \langle\nabla_6\rangle, \langle\nabla_6+\nabla_7\rangle, \langle\nabla_7\rangle,$$

which gives the following new algebras:

$\mathbf{N}_{314}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$	
		$e_2e_2=\alpha e_5$	$e_2e_3=\beta e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{315}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=\alpha e_5$	
		$e_2e_2=\beta e_5$	$e_2e_3=\gamma e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{316}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_2e_2=\alpha e_5$	$e_2e_3=\beta e_5$
		$e_2e_4=\gamma e_5$	$e_3e_3=e_4$	$e_3e_4=\mu e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{317}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_3e_3=e_4$	

$\mathbf{N}_{318}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$		
		$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{319}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=\alpha e_5$	
		$e_2e_3=\beta e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{320}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=\alpha e_5$	
		$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{321}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_3=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{322}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$		
		$e_2e_3=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{323}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	
		$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{324}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	
		$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{325}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$		
		$e_2e_4=e_5$	$e_3e_3=e_4$			
$\mathbf{N}_{326}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$		
		$e_2e_4=\alpha e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{327}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$	$e_2e_4=\alpha e_5$	
		$e_3e_3=e_4$	$e_3e_4=\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{328}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{329}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{330}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_4=e_5$		
		$e_3e_3=e_4$	$e_3e_4=\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{331}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{332}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_3e_3=e_4$	$e_3e_4=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{333}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_3e_3=e_4$	$e_4e_4=e_5$	

4.4. **1-dimensional central extensions of $\mathbf{N}_{04}^4$.** Here we will collect all information about $\mathbf{N}_{04}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{04}^4$	$e_1e_1=e_2$ $e_1e_2=e_3$ $e_2e_2=e_4$ $e_3e_3=e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{04}^4)=\Big\langle[\Delta_{ij}]\Big\rangle$ $(i,j)\notin\{(1,1),(1,2),(3,3)\}$	$\phi_\pm=\begin{pmatrix}\pm1&0&0&0\\0&1&0&0\\0&0&\pm1&0\\t&0&0&1\end{pmatrix}$

Let us use the following notations:

$$\nabla_1=[\Delta_{13}],\quad \nabla_2=[\Delta_{14}],\quad \nabla_3=[\Delta_{22}],\quad \nabla_4=[\Delta_{23}],$$
$$\nabla_5=[\Delta_{24}],\quad \nabla_6=[\Delta_{34}],\quad \nabla_7=[\Delta_{44}].$$

Take $\theta=\sum\limits_{i=1}^{7}\alpha_i\nabla_i\in\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{04}^4)$. Since

$$\phi^T\begin{pmatrix}0&0&\alpha_1&\alpha_2\\0&\alpha_3&\alpha_4&\alpha_5\\\alpha_1&\alpha_4&0&\alpha_6\\\alpha_2&\alpha_5&\alpha_6&\alpha_7\end{pmatrix}\phi=\begin{pmatrix}\alpha^*&\alpha^{**}&\alpha_1^*&\alpha_2^*\\\alpha^{**}&\alpha_3^*&\alpha_4^*&\alpha_5^*\\\alpha_1^*&\alpha_4^*&0&\alpha_6^*\\\alpha_2^*&\alpha_5^*&\alpha_6^*&\alpha_7^*\end{pmatrix}$$

we have

$$\begin{array}{llll}\alpha_1^* = \alpha_1 \pm \alpha_6 t, & \alpha_2^* = \pm\alpha_2 + \alpha_7 t, & \alpha_3^* = \alpha_3, & \alpha_4^* = \pm\alpha_4,\\ \alpha_5^* = \alpha_5, & \alpha_6^* = \pm\alpha_6, & \alpha_7^* = \alpha_7. &\end{array}$$

We are interested in $(\alpha_2, \alpha_5, \alpha_6, \alpha_7) \neq (0,0,0,0)$ and consider following cases:

1. if $\alpha_7 = \alpha_6 = \alpha_5 = 0$, then $\alpha_2 \neq 0$, and we have the family of representatives
$$\langle \alpha\nabla_1 + \nabla_2 + \beta\nabla_3 + \gamma\nabla_4 \rangle;$$
2. if $\alpha_7 = 0, \alpha_6 = 0, \alpha_5 \neq 0$, then we have the family of representatives
$$\langle \alpha\nabla_1 + \beta\nabla_2 + \gamma\nabla_3 + \mu\nabla_4 + \nabla_5 \rangle;$$
3. if $\alpha_7 = 0, \alpha_6 \neq 0$, then by choosing $\phi = \phi_+, t = -\alpha_1\alpha_6^{-1}$, we have the family of representatives
$$\langle \alpha\nabla_2 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_6 \rangle;$$
4. if $\alpha_7 \neq 0$, then by choosing $\phi = \phi_+, t = -\alpha_2\alpha_7^{-1}$ we have the family of representatives
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nu\nabla_6 + \nabla_7 \rangle.$$

Summarizing, we have the following distinct orbits:

$$\langle \alpha\nabla_1 + \nabla_2 + \beta\nabla_3 + \gamma\nabla_4 \rangle^{O(\alpha,\beta,\gamma)=O(-\alpha,-\beta,\gamma)},$$
$$\langle \alpha\nabla_1 + \beta\nabla_2 + \gamma\nabla_3 + \mu\nabla_4 + \nabla_5 \rangle^{O(\alpha,\beta,\gamma,\mu)=O(\alpha,-\beta,\gamma,-\mu)}$$
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nu\nabla_6 + \nabla_7 \rangle^{O(\alpha,\beta,\gamma,\mu,\nu)=O(\alpha,\beta,-\gamma,\mu,-\nu)},$$
$$\langle \alpha\nabla_2 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_6 \rangle^{O(\alpha,\beta,\gamma,\mu)=O(\alpha,-\beta,\gamma,-\mu)},$$

which gives the following new algebras:

$\mathbf{N}_{334}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	$e_1e_4 = e_5$
		$e_2e_2 = e_4 + \beta e_5$	$e_2e_3 = \gamma e_5$	$e_3e_3 = e_4$	
$\mathbf{N}_{335}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	$e_1e_4 = \beta e_5$
		$e_2e_2 = e_4 + \gamma e_5$	$e_2e_3 = \mu e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{336}^{\alpha,\beta,\gamma,\mu,\nu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	
		$e_2e_2 = e_4 + \beta e_5$	$e_2e_3 = \gamma e_5$	$e_2e_4 = \mu e_5$	
		$e_3e_3 = e_4$	$e_3e_4 = \nu e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{337}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_4 + \beta e_5$
		$e_2e_3 = \gamma e_5$	$e_2e_4 = \mu e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$

4.5. **1-dimensional central extensions of $\mathbf{N}_{05}^4$.** Here we will collect all information about $\mathbf{N}_{05}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{05}^4$	$e_1e_1 = e_2$ $e_1e_3 = e_4$ $e_2e_2 = e_3$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{05}^4) = \Big\langle [\Delta_{ij}] \Big\rangle$ $(i,j) \notin \{(1,1),(1,3),(2,2)\}$	$\phi = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & x^2 & 0 & 0 \\ z & 0 & x^4 & 0 \\ t & 2xz & 0 & x^5 \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll}\nabla_1 = [\Delta_{12}] & \nabla_2 = [\Delta_{14}] & \nabla_3 = [\Delta_{23}] & \nabla_4 = [\Delta_{24}]\\ \nabla_5 = [\Delta_{33}] & \nabla_6 = [\Delta_{34}] & \nabla_7 = [\Delta_{44}]. &\end{array}$$

Take $\theta = \sum\limits_{i=1}^{7} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^4_{05})$. Since

$$\phi^T \begin{pmatrix} 0 & \alpha_1 & 0 & \alpha_2 \\ \alpha_1 & 0 & \alpha_3 & \alpha_4 \\ 0 & \alpha_3 & \alpha_5 & \alpha_6 \\ \alpha_2 & \alpha_4 & \alpha_6 & \alpha_7 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha_1^* & \alpha^{***} & \alpha_2^* \\ \alpha_1^* & \alpha^{**} & \alpha_3^* & \alpha_4^* \\ \alpha^{***} & \alpha_3^* & \alpha_5^* & \alpha_6^* \\ \alpha_2^* & \alpha_4^* & \alpha_6^* & \alpha_7^* \end{pmatrix}$$

we have

$$\begin{array}{lll} \alpha_1^* = (\alpha_1 x + \alpha_3 z + \alpha_4 t)x^2 + 2(\alpha_2 x + \alpha_6 z + \alpha_7 t)xz, & & \\ \alpha_2^* = (\alpha_2 x + \alpha_6 z + \alpha_7 t)x^5, & \alpha_3^* = (\alpha_3 x + 2\alpha_6 z)x^5, & \alpha_4^* = (\alpha_4 x + 2\alpha_7 z)x^6, \\ \alpha_5^* = \alpha_5 x^8, & \alpha_6^* = \alpha_6 x^9, & \alpha_7^* = \alpha_7 x^{10}. \end{array}$$

We are interested in $(\alpha_2, \alpha_4, \alpha_6, \alpha_7) \neq (0,0,0,0)$ and consider following cases:

1. $\alpha_7 = \alpha_6 = \alpha_4 = 0$, then $\alpha_2 \neq 0$ and we have the following subcases:
   (a) $\alpha_3 = -2\alpha_2$,
      (i) if $\alpha_1 = 0, \alpha_5 = 0$, then we have the representative $\langle \nabla_2 - 2\nabla_3 \rangle$;
      (ii) if $\alpha_1 = 0, \alpha_5 \neq 0$, then by choosing $x = \sqrt{\alpha_2 \alpha_5^{-1}}, z = 0, t = 0$, we have the representative $\langle \nabla_2 - 2\nabla_3 + \nabla_5 \rangle$;
      (iii) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_1 \alpha_2^{-1}}, z = 0, t = 0$, we have the family of representatives $\langle \nabla_1 + \nabla_2 - 2\nabla_3 + \alpha\nabla_5 \rangle$.
   (b) $\alpha_3 \neq -2\alpha_2$,
      (i) if $\alpha_5 = 0$, then choosing $x = 1, z = -\frac{\alpha_1}{\alpha_3 + 2\alpha_2}$, we have the family of representatives $\langle \nabla_2 + \alpha\nabla_3 \rangle_{\alpha \neq -2}$, which will be jointed with the case (1(a)i);
      (ii) if $\alpha_5 \neq 0$, then by choosing $x = \sqrt{\frac{\alpha_2}{\alpha_5}}, z = -\frac{\alpha_1 \sqrt{\alpha_2}}{(\alpha_3 + 2\alpha_2)\sqrt{\alpha_5}}$, we have the family of representatives $\langle \nabla_2 + \alpha\nabla_3 + \nabla_5 \rangle_{\alpha \neq -2}$, which will be jointed with the case (1(a)ii).
2. $\alpha_7 = \alpha_6 = 0, \alpha_4 \neq 0$, then we have the following subcases:
   (a) if $\alpha_2 = 0, \alpha_3 = 0$, then we have representatives $\langle \nabla_4 \rangle$ and $\langle \nabla_4 + \nabla_5 \rangle$ depending on whether $\alpha_5 = 0$ or not;
   (b) if $\alpha_2 = 0, \alpha_3 \neq 0$, then by choosing $x = \alpha_3 \alpha_4^{-1}, z = 0, t = -\alpha_1 \alpha_3 \alpha_4^{-2}$, we have the family of representatives $\langle \nabla_3 + \nabla_4 + \alpha\nabla_5 \rangle$;
   (c) if $\alpha_2 \neq 0$, then by choosing $x = \alpha_2 \alpha_4^{-1}, z = 0, t = -\alpha_1 \alpha_2 \alpha_4^{-2}$, we have the family of representatives $\langle \nabla_2 + \alpha\nabla_3 + \nabla_4 + \beta\nabla_5 \rangle$.
3. $\alpha_7 = 0, \alpha_6 \neq 0$, then by choosing $z = -\alpha_2 x \alpha_6^{-1}$, we have $\alpha_2^* = 0$. Thus, we can suppose that $\alpha_2 = 0$ and consider following subcases:
   (a) $\alpha_4 = 0$,
      (i) if $\alpha_1 = 0, \alpha_3 = 0$, then we have representatives $\langle \nabla_6 \rangle$ and $\langle \nabla_5 + \nabla_6 \rangle$ depending on whether $\alpha_5 = 0$ or not;
      (ii) if $\alpha_1 = 0, \alpha_3 \neq 0$ then by choosing $x = \sqrt[3]{\alpha_3 \alpha_6^{-1}}, z = 0, t = 0$, we have the family of representatives $\langle \nabla_3 + \alpha\nabla_5 + \nabla_6 \rangle$;
      (iii) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[6]{\alpha_1 \alpha_6^{-1}}, z = 0, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_5 + \nabla_6 \rangle$.
   (b) $\alpha_4 \neq 0$, then by choosing $x = \sqrt{\alpha_4 \alpha_6^{-1}}, t = -\alpha_1 \sqrt{\alpha_4 \alpha_6^{-3}}$, we have the family of representatives $\langle \alpha\nabla_3 + \nabla_4 + \beta\nabla_5 + \nabla_6 \rangle$.
4. $\alpha_7 \neq 0$,

(a) if $\alpha_1\alpha_7-\alpha_2\alpha_4=0, \alpha_3=0, \alpha_6=0$, then then by choosing $z=-\frac{\alpha_4}{2\alpha_7}x, t=\frac{\alpha_4\alpha_6-2\alpha_2\alpha_7}{2\alpha_7^2}x$, we have representatives $\langle\nabla_7\rangle$ and $\langle\nabla_5+\nabla_7\rangle$ depending on whether $\alpha_5=0$ or not;

(b) if $\alpha_1\alpha_7-\alpha_2\alpha_4=0, \alpha_3\alpha_7-\alpha_4\alpha_6=0, \alpha_6\neq 0$, then by choosing
$$x=\frac{\alpha_6}{\alpha_7}, z=-\frac{\alpha_4\alpha_6}{2\alpha_7^2}, t=\frac{\alpha_6(\alpha_4\alpha_6-2\alpha_2\alpha_7)}{2\alpha_7^3},$$
we have the family of representatives $\langle\alpha\nabla_5+\nabla_6+\nabla_7\rangle$;

(c) if $2\alpha_1\alpha_7^2-\alpha_3\alpha_4\alpha_7+\alpha_4^2\alpha_6-2\alpha_2\alpha_4\alpha_7=0, \alpha_3\alpha_7-\alpha_4\alpha_6\neq 0$, then by choosing
$$x=\sqrt[4]{\frac{\alpha_3\alpha_7-\alpha_4\alpha_7}{\alpha_7^2}}, z=-\frac{\alpha_4\sqrt[4]{\alpha_3\alpha_7-\alpha_4\alpha_7}}{2\alpha_7\sqrt[4]{2\alpha_7^2}}, t=\frac{(\alpha_4\alpha_6-2\alpha_2\alpha_7)\sqrt[4]{\alpha_3\alpha_7-\alpha_4\alpha_6}}{2\alpha_7^2\sqrt[4]{2\alpha_7^2}},$$
we have the family of representatives $\langle\nabla_3+\alpha\nabla_5+\beta\nabla_6+\nabla_7\rangle$;

(d) if $2\alpha_1\alpha_7^2-\alpha_3\alpha_4\alpha_7+\alpha_4^2\alpha_6-2\alpha_2\alpha_4\alpha_7\neq 0$, then by choosing
$$x=\sqrt[7]{\frac{2\alpha_1\alpha_7^2-\alpha_3\alpha_4\alpha_7+\alpha_4^2\alpha_6-2\alpha_2\alpha_4\alpha_7}{2\alpha_7^3}},$$
$$z=-\frac{\alpha_4\sqrt[7]{2\alpha_1\alpha_7^2-\alpha_3\alpha_4\alpha_7+\alpha_4^2\alpha_6-2\alpha_2\alpha_4\alpha_7}}{2\alpha_7\sqrt[7]{2\alpha_7^3}},$$
$$t=\frac{(\alpha_4\alpha_6-2\alpha_2\alpha_7)\sqrt[7]{2\alpha_1\alpha_7^2-\alpha_3\alpha_4\alpha_7+\alpha_4^2\alpha_6-2\alpha_2\alpha_4\alpha_7}}{2\alpha_7^3\sqrt[7]{2\alpha_7^3}},$$
we have the family of representatives $\langle\nabla_1+\alpha\nabla_3+\beta\nabla_5+\gamma\nabla_6+\nabla_7\rangle$.

Summarizing, we have the following distinct orbits:
$$\langle\nabla_1+\nabla_2-2\nabla_3+\alpha\nabla_5\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},$$
$$\langle\nabla_1+\alpha\nabla_3+\beta\nabla_5+\nabla_6\rangle^{\substack{O(\alpha,\beta)=O(\alpha,-\eta_3\beta)=O(-\alpha,\eta_3\beta)=\\ O(-\alpha,-\eta_3^2\beta)=O(\alpha,\eta_3^2\beta)=O(-\alpha,-\beta)}},$$
$$\langle\nabla_1+\alpha\nabla_3+\beta\nabla_5+\gamma\nabla_6+\nabla_7\rangle^{\substack{O(\alpha,\beta,\gamma)=O(\eta_7^4\alpha,\eta_7^2\beta,-\eta_7\gamma)=O(-\eta_7\alpha,\eta_7^4\beta,\eta_7^2\gamma)=\\ O(-\eta_7^5\alpha,\eta_7^6\beta,-\eta_7^3\gamma)=O(\eta_7^2\alpha,-\eta_7\beta,\eta_7^4\gamma)=\\ O(\eta_7^6\alpha,-\eta_7^3\beta,-\eta_7^5\gamma)=O(-\eta_7^3\alpha,-\eta_7^5\beta,\eta_7^6\gamma)}},$$
$$\langle\nabla_2+\alpha\nabla_3\rangle, \langle\nabla_2+\alpha\nabla_3+\nabla_4+\beta\nabla_5\rangle, \langle\nabla_2+\alpha\nabla_3+\nabla_5\rangle, \langle\nabla_3+\nabla_4+\alpha\nabla_5\rangle,$$
$$\langle\alpha\nabla_3+\nabla_4+\beta\nabla_5+\nabla_6\rangle^{O(\alpha,\beta)=O(-\alpha,-\beta)}, \langle\nabla_3+\alpha\nabla_5+\nabla_6\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},$$
$$\langle\nabla_3+\alpha\nabla_5+\beta\nabla_6+\nabla_7\rangle^{O(\alpha,\beta)=O(-\alpha,-i\beta)=O(-\alpha,i\beta)=O(\alpha,-\beta)}, \langle\nabla_4\rangle, \langle\nabla_4+\nabla_5\rangle,$$
$$\langle\nabla_5+\nabla_6\rangle, \langle\alpha\nabla_5+\nabla_6+\nabla_7\rangle, \langle\nabla_5+\nabla_7\rangle, \langle\nabla_6\rangle, \langle\nabla_7\rangle,$$
which gives the following new algebras:

$\mathbf{N}_{338}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_4$	$e_1e_4=e_5$	
		$e_2e_2=e_3$	$e_2e_3=-2e_5$	$e_3e_3=\alpha e_5$		
$\mathbf{N}_{339}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_4$	$e_2e_2=e_3$	
		$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{340}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_4$	$e_2e_2=e_3$	
		$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{341}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=\alpha e_5$
$\mathbf{N}_{342}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=e_5$	$e_2e_2=e_3$	
		$e_2e_3=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=\beta e_5$		
$\mathbf{N}_{343}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=e_5$		
		$e_2e_2=e_3$	$e_2e_3=\alpha e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{344}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$		
		$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$		
$\mathbf{N}_{345}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_3=\alpha e_5$	
		$e_2e_4=e_5$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{346}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$		
		$e_2e_3=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{347}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_3=e_5$	

		$e_3e_3=\alpha e_5$	$e_3e_4=\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{348}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_4=e_5$	
$\mathbf{N}_{349}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{350}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_3e_3=e_5$	$e_3e_4=e_5$
$\mathbf{N}_{351}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$		
		$e_3e_3=\alpha e_5$	$e_3e_4=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{352}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{353}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_3e_4=e_5$	
$\mathbf{N}_{354}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_4e_4=e_5$	

4.6. **1-dimensional central extensions of $\mathbf{N}_{06}^4$.** Here we will collect all information about $\mathbf{N}_{06}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{06}^4$	$e_1e_1=e_2$ $e_1e_2=e_4$ $e_1e_3=e_4$ $e_2e_2=e_3$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{06}^4)=\Big\langle[\Delta_{ij}]\Big\rangle$ $(i,j)\notin\{(1,1),(1,2),(2,2)\}$	$\phi_{\pm}=\begin{pmatrix}\pm1&0&0&0\\0&1&0&0\\z&0&1&0\\t&\pm2z&0&\pm1\end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll}\nabla_1=[\Delta_{13}], & \nabla_2=[\Delta_{14}], & \nabla_3=[\Delta_{23}], & \nabla_4=[\Delta_{24}],\\ \nabla_5=[\Delta_{33}], & \nabla_6=[\Delta_{34}], & \nabla_7=[\Delta_{44}].\end{array}$$

Take $\theta=\sum\limits_{i=1}^{7}\alpha_i\nabla_i\in\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{06}^4)$. Since

$$\phi_{\pm}^T\begin{pmatrix}0&0&\alpha_1&\alpha_2\\0&0&\alpha_3&\alpha_4\\\alpha_1&\alpha_3&\alpha_5&\alpha_6\\\alpha_2&\alpha_4&\alpha_6&\alpha_7\end{pmatrix}\phi_{\pm}=\begin{pmatrix}\alpha^*&\alpha^{**}&\alpha_1^*+\alpha^{**}&\alpha_2^*\\\alpha^{**}&\alpha^{***}&\alpha_3^*&\alpha_4^*\\\alpha_1^*+\alpha^{**}&\alpha_3^*&\alpha_5^*&\alpha_6^*\\\alpha_2^*&\alpha_4^*&\alpha_6^*&\alpha_7^*\end{pmatrix}$$

we have

$$\begin{array}{lll}\alpha_1^*=\pm\alpha_1-\alpha_3z-\alpha_4t+\alpha_5z+\alpha_6t-2(\alpha_2\pm\alpha_6z\pm\alpha_7t)z, & & \\ \alpha_2^*=\alpha_2\pm\alpha_6z\pm\alpha_7t, & \alpha_3^*=\alpha_3\pm2\alpha_6z, & \alpha_4^*=2\alpha_7z\pm\alpha_4,\\ \alpha_5^*=\alpha_5, & \alpha_6^*=\pm\alpha_6, & \alpha_7^*=\alpha_7.\end{array}$$

Since $(\alpha_2,\alpha_4,\alpha_6,\alpha_7)\neq(0,0,0,0)$ and for $\phi=\phi_+$, we have the following cases:

1. if $\alpha_7=\alpha_6=\alpha_4=0$, then $\alpha_2\neq0$, and we have the following subcase:
   (a) if $\alpha_5=\alpha_3+2\alpha_2$, then we have the family of representatives $$\langle\alpha\nabla_1+\nabla_2+\beta\nabla_3+(\beta+2)\nabla_5\rangle;$$
   (b) if $\alpha_5\neq\alpha_3+2\alpha_2$, then by choosing $z=0, t=\frac{\alpha_1}{\alpha_3+2\alpha_2-\alpha_5}$, we have the family of representative $\langle\nabla_2+\alpha\nabla_3+\beta\nabla_5\rangle_{\beta\neq\alpha+2}$;
2. if $\alpha_7=0,\alpha_6=0,\alpha_4\neq0$, then by choosing $z=0,t=\frac{\alpha_1}{\alpha_4}$, we have the family of representatives $\langle\alpha\nabla_2+\beta\nabla_3+\nabla_4+\gamma\nabla_5\rangle$;
3. if $\alpha_7=0,\alpha_6\neq0$, then we have the following subcases:
   (a) if $\alpha_6=\alpha_4$, then by choosing $z=-\alpha_2\alpha_6^{-1}, t=0$, we have the family of representatives $\langle\alpha\nabla_1+\beta\nabla_3+\nabla_4+\gamma\nabla_5+\nabla_6\rangle$;
   (b) if $\alpha_6\neq\alpha_4$, then by choosing $z=-\frac{\alpha_2}{\alpha_6}, t=\frac{\alpha_1\alpha_6-\alpha_2\alpha_5+\alpha_2\alpha_3}{\alpha_6(\alpha_4-\alpha_6)}$, we have the family of representatives $\langle\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\nabla_6\rangle_{\beta\neq1}$;

4. if $\alpha_7 \neq 0$, then by choosing $z = -\frac{\alpha_4}{2\alpha_7}, t = \frac{\alpha_4\alpha_6 - 2\alpha_2\alpha_7}{2\alpha_7^2}$, we have the family of representatives
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_5 + \mu\nabla_6 + \nabla_7 \rangle.$$

Summarizing, we have the following distinct orbits:
$$\langle \alpha\nabla_1 + \nabla_2 + \beta\nabla_3 + (\beta+2)\nabla_5 \rangle_{\alpha\neq 0}^{O(\alpha,\beta)=O(-\alpha,\beta)},$$
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \nabla_4 + \gamma\nabla_5 + \nabla_6 \rangle_{\alpha\neq 0}^{O(\alpha,\beta,\gamma)=O(\alpha,-\beta,-\gamma)},$$
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_5 + \mu\nabla_6 + \nabla_7 \rangle^{O(\alpha,\beta,\gamma,\mu)=O(-\alpha,\beta,\gamma,-\mu)}, \ \langle \nabla_2 + \alpha\nabla_3 + \beta\nabla_5 \rangle,$$
$$\langle \alpha\nabla_2 + \beta\nabla_3 + \nabla_4 + \gamma\nabla_5 \rangle^{O(\alpha,\beta,\gamma)=O(-\alpha,-\beta,-\gamma)},$$
$$\langle \alpha\nabla_3 + \beta\nabla_4 + \gamma\nabla_5 + \nabla_6 \rangle^{O(\alpha,\beta,\gamma)=O(-\alpha,\beta,-\gamma)},$$
which gives the following new algebras:

$\mathbf{N}_{355}^{\alpha\neq 0,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4 + \alpha e_5$	$e_1e_4 = e_5$	
		$e_2e_2 = e_3$	$e_2e_3 = \beta e_5$	$e_3e_3 = (\beta+2)e_5$	
$\mathbf{N}_{356}^{\alpha\neq 0,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4 + \alpha e_5$	$e_2e_2 = e_3$	$e_2e_3 = \beta e_5$
		$e_2e_4 = e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{357}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4 + \alpha e_5$	$e_2e_2 = e_3$	$e_2e_3 = \beta e_5$
		$e_3e_3 = \gamma e_5$	$e_3e_4 = \mu e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{358}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{359}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_3$
		$e_2e_3 = \beta e_5$	$e_2e_4 = e_5$	$e_3e_3 = \gamma e_5$	
$\mathbf{N}_{360}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$
		$e_2e_4 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = e_5$	

4.7. **1-dimensional central extensions of $\mathbf{N}_{07}^4$.** Here we will collect all information about $\mathbf{N}_{07}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{07}^4$	$e_1e_1 = e_2$ $e_2e_2 = e_3$ $e_2e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{07}^4) = \Big\langle [\Delta_{ij}] \Big\rangle$ $(i,j) \notin \{(1,1),(2,2),(2,3)\}$	$\phi = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & x^2 & 0 & 0 \\ 0 & 0 & x^4 & 0 \\ t & 0 & 0 & x^6 \end{pmatrix}$

Let us use the following notations:
$$\nabla_1 = [\Delta_{12}], \quad \nabla_2 = [\Delta_{13}], \quad \nabla_3 = [\Delta_{14}], \quad \nabla_4 = [\Delta_{24}],$$
$$\nabla_5 = [\Delta_{33}], \quad \nabla_6 = [\Delta_{34}], \quad \nabla_7 = [\Delta_{44}].$$

Take $\theta = \sum\limits_{i=1}^{7} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{07}^4)$. Since
$$\phi^T \begin{pmatrix} 0 & \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_1 & 0 & 0 & \alpha_4 \\ \alpha_2 & 0 & \alpha_5 & \alpha_6 \\ \alpha_3 & \alpha_4 & \alpha_6 & \alpha_7 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha_1^* & \alpha_2^* & \alpha_3^* \\ \alpha_1^* & 0 & 0 & \alpha_4^* \\ \alpha_2^* & 0 & \alpha_5^* & \alpha_6^* \\ \alpha_3^* & \alpha_4^* & \alpha_6^* & \alpha_7^* \end{pmatrix}$$
we have
$$\begin{array}{llll} \alpha_1^* = (\alpha_1 x + \alpha_4 t)x^2, & \alpha_2^* = (\alpha_2 x + \alpha_6 t)x^4, & \alpha_3^* = (\alpha_3 x + \alpha_7 t)x^6, & \alpha_4^* = \alpha_4 x^8, \\ \alpha_5^* = \alpha_5 x^8, & \alpha_6^* = \alpha_6 x^{10}, & \alpha_7^* = \alpha_7 x^{12}. & \end{array}$$

We are interested in $(\alpha_3, \alpha_4, \alpha_6, \alpha_7) \neq (0,0,0,0)$ and consider following cases:

1. $\alpha_7 = \alpha_6 = \alpha_4 = 0$, then $\alpha_3 \neq 0$, and we have the following subcases:
   (a) if $\alpha_1 = 0, \alpha_2 = 0$, then we have representatives $\langle \nabla_3 \rangle$ and $\langle \nabla_3 + \nabla_5 \rangle$ depending on whether $\alpha_5 = 0$ or not;
   (b) if $\alpha_1 = 0, \alpha_2 \neq 0$, then by choosing $x = \sqrt{\alpha_2 \alpha_3^{-1}}, t = 0$, we have the family of representatives $\langle \nabla_2 + \nabla_3 + \alpha \nabla_5 \rangle$;
   (c) if $\alpha_1 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_1 \alpha_3^{-1}}, t = 0$, we have the family of representatives $\langle \nabla_1 + \alpha \nabla_2 + \nabla_3 + \beta \nabla_5 \rangle$.
2. $\alpha_7 = 0, \alpha_6 = 0, \alpha_4 \neq 0$, then we have the following subcases:
   (a) if $\alpha_2 = 0, \alpha_3 = 0$, then we have the familty of representative $\langle \nabla_4 + \alpha \nabla_5 \rangle$;
   (b) if $\alpha_2 = 0, \alpha_3 \neq 0$, then by choosing $x = \alpha_3 \alpha_4^{-1}, t = -\alpha_1 \alpha_3 \alpha_4^{-2}$, we have the family of representatives $\langle \nabla_3 + \nabla_4 + \alpha \nabla_5 \rangle$;
   (c) if $\alpha_2 \neq 0$, then by choosing $x = \sqrt[3]{\alpha_2 \alpha_4^{-1}}, t = -\alpha_1 \sqrt[3]{\alpha_2 \alpha_4^{-4}}$, we have the family of representatives $\langle \nabla_2 + \alpha \nabla_3 + \nabla_4 + \beta \nabla_5 \rangle$.
3. $\alpha_7 = 0, \alpha_6 \neq 0$, then we have the following subcases:
   (a) if $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0$, then we have representatives $\langle \nabla_6 \rangle$ and $\langle \nabla_5 + \nabla_6 \rangle$ depending on whether $\alpha_5 = 0$ or not;
   (b) if $\alpha_1 \alpha_6 = \alpha_2 \alpha_4, \alpha_3 = 0, \alpha_4 \neq 0$, then choosing $x = \sqrt{\alpha_4 \alpha_6^{-1}}, t = -\alpha_2 \sqrt{\alpha_4} \alpha_6^{-3}$, we have the family of representatives $\langle \nabla_4 + \alpha \nabla_5 + \nabla_6 \rangle$;
   (c) if $\alpha_1 \alpha_6 = \alpha_2 \alpha_4, \alpha_3 \neq 0$, then by choosing $x = \sqrt[3]{\frac{\alpha_3}{\alpha_6}}, t = -\frac{\alpha_2 \sqrt[3]{\alpha_3}}{\alpha_6 \sqrt[3]{\alpha_6}}$, we have the family of representatives $\langle \nabla_3 + \alpha \nabla_4 + \beta \nabla_5 + \nabla_6 \rangle$;
   (d) if $\alpha_1 \alpha_6 \neq \alpha_2 \alpha_4$, then choosing $x = \sqrt[7]{\frac{\alpha_1 \alpha_6 - \alpha_2 \alpha_4}{\alpha_6^2}}, t = -\frac{\alpha_2 \sqrt[7]{\alpha_1 \alpha_6 - \alpha_2 \alpha_4}}{\alpha_6 \sqrt[7]{\alpha_6^2}}$, we have the family of representatives $\langle \nabla_1 + \alpha \nabla_3 + \beta \nabla_4 + \gamma \nabla_5 + \nabla_6 \rangle$.
4. $\alpha_7 \neq 0$, then we have the following subcases:
   (a) $\alpha_1 = 0, \alpha_2 \alpha_7 = \alpha_3 \alpha_6, \alpha_4 = 0, \alpha_5 = 0$, then we have representatives $\langle \nabla_7 \rangle$ and $\langle \nabla_6 + \nabla_7 \rangle$ depending on whether $\alpha_6 = 0$ or not;
   (b) $\alpha_1 = 0, \alpha_2 \alpha_7 = \alpha_3 \alpha_6, \alpha_4 = 0, \alpha_5 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_5 \alpha_7^{-1}}, t = -\alpha_3 \sqrt[4]{\alpha_5 \alpha_7^{-5}}$, we have family of representatives $\langle \nabla_5 + \alpha \nabla_6 + \nabla_7 \rangle$;
   (c) $\alpha_1 \alpha_7 = \alpha_3 \alpha_4, \alpha_2 \alpha_7 = \alpha_3 \alpha_6, \alpha_4 \neq 0$, then by choosing $x = \sqrt[4]{\alpha_4 \alpha_7^{-1}}, t = -\alpha_3 \sqrt[4]{\alpha_4 \alpha_7^{-5}}$, we have family of representatives $\langle \nabla_4 + \alpha \nabla_5 + \beta \nabla_6 + \nabla_7 \rangle$;
   (d) $\alpha_1 \alpha_7 = \alpha_3 \alpha_4, \alpha_2 \alpha_7 \neq \alpha_3 \alpha_6$, then by choosing $x = \sqrt[7]{(\alpha_2 \alpha_7 - \alpha_3 \alpha_6) \alpha_7^{-2}}$, $t = -\alpha_3 \sqrt[7]{(\alpha_2 \alpha_7 - \alpha_3 \alpha_6) \alpha_7^{-9}}$, we have the family of representatives
   $$\langle \nabla_2 + \alpha \nabla_4 + \beta \nabla_5 + \gamma \nabla_6 + \nabla_7 \rangle;$$
   (e) $\alpha_1 \alpha_7 \neq \alpha_3 \alpha_4$, then by choosing
   $$x = \sqrt[9]{(\alpha_1 \alpha_7 - \alpha_3 \alpha_4) \alpha_7^{-2}}, t = -\alpha_3 \sqrt[9]{(\alpha_1 \alpha_7 - \alpha_3 \alpha_4) \alpha_7^{-11}},$$
   we have family of representatives $\langle \nabla_1 + \alpha \nabla_2 + \beta \nabla_4 + \gamma \nabla_5 + \mu \nabla_6 + \nabla_7 \rangle$.

Summarizing, we have the following distinct orbits:

$$\langle \nabla_1 + \alpha \nabla_2 + \nabla_3 + \beta \nabla_5 \rangle^{O(\alpha,\beta)=O(-\alpha,i\beta)=O(-\alpha,-i\beta)=O(\alpha,-\beta)},$$

$$\langle \nabla_1 + \alpha \nabla_2 + \beta \nabla_4 + \gamma \nabla_5 + \mu \nabla_6 + \nabla_7 \rangle^{\substack{O(\alpha,\beta,\gamma,\mu) = O(-\eta_9^7 \alpha, \eta_9^4 \beta, \eta_9^4 \gamma, \eta_9^2 \mu) = \\ O(-\eta_9^5 \alpha, \eta_9^8 \beta, \eta_9^8 \gamma, \eta_9^4 \mu) = O(-\eta_3 \alpha, -\eta_3 \beta, -\eta_3 \gamma, \eta_3^2 \mu) = \\ O(-\eta_9 \alpha, -\eta_9^7 \beta, -\eta_9^7 \gamma, \eta_9^8 \mu) = O(\eta_9^8 \alpha, \eta_9^2 \beta, \eta_9^2 \gamma, -\eta_9 \mu) = \\ O(\eta_3^2 \alpha, \eta_3^2 \beta, \eta_3^2 \gamma, -\eta_3 \mu) = O(\eta_9^4 \alpha, -\eta_9 \beta, -\eta_9 \gamma, -\eta_9^5 \mu) = \\ O(\eta_9^2 \alpha, -\eta_9^5 \beta, -\eta_9^5 \gamma, -\eta_9^7 \mu)}},$$

$$\langle \nabla_1 + \alpha\nabla_3 + \beta\nabla_4 + \gamma\nabla_5 + \nabla_6 \rangle^{\substack{O(\alpha,\beta,\gamma) = O(-\eta_7^3\alpha, \eta_7^2\beta, \eta_7^2\gamma) = \\ O(\eta_7^6\alpha, \eta_7^4\beta, \eta_7^4\gamma) = O(\eta_7^2\alpha, \eta_7^6\beta, \eta_7^6\gamma) = \\ O(-\eta_7^5\alpha, -\eta_7\beta, -\eta_7\gamma) = O(-\eta_7\alpha, -\eta_7^3\beta, -\eta_7^3\gamma) = \\ O(\eta_7^4\alpha, -\eta_7^5\beta, -\eta_7^5\gamma)}},$$

$$\langle \nabla_2 + \alpha\nabla_3 + \nabla_4 + \beta\nabla_5 \rangle^{O(\alpha,\beta)=O(-\eta_3\alpha,\beta)=O(\eta_3^2\alpha,\beta)}, \ \langle \nabla_2 + \nabla_3 + \alpha\nabla_5 \rangle^{O(\alpha)=O(-\alpha)},$$

$$\langle \nabla_2 + \alpha\nabla_4 + \beta\nabla_5 + \gamma\nabla_6 + \nabla_7 \rangle^{\substack{O(\alpha,\beta,\gamma) = O(\eta_7^4\alpha, \eta_7^4\beta, \eta_7^2\gamma) = O(-\eta_7\alpha, -\eta_7\beta, \eta_7^4\gamma) = \\ O(-\eta_7^5\alpha, -\eta_7^5\beta, \eta_7^6\gamma) = O(\eta_7^2\alpha, \eta_7^2\beta, -\eta_7\gamma) = \\ O(\eta_7^6\alpha, \eta_7^6\beta, -\eta_7^3\gamma) = O(-\eta_7^3\alpha, -\eta_7^3\beta, -\eta_7^5\gamma)}}, \ \langle \nabla_3 \rangle,$$

$$\langle \nabla_3 + \nabla_4 + \alpha\nabla_5 \rangle, \ \langle \nabla_3 + \alpha\nabla_4 + \beta\nabla_5 + \nabla_6 \rangle^{O(\alpha,\beta)=O(-\eta_3\alpha,-\eta_3\beta)=O(\eta_3^2\alpha,\eta_3^2\beta)},$$

$$\langle \nabla_3 + \nabla_5 \rangle, \ \langle \nabla_4 + \alpha\nabla_5 \rangle, \ \langle \nabla_4 + \alpha\nabla_5 + \nabla_6 \rangle, \ \langle \nabla_4 + \alpha\nabla_5 + \beta\nabla_6 + \nabla_7 \rangle^{O(\alpha,\beta)=O(\alpha,-\beta)},$$

$$\langle \nabla_5 + \nabla_6 \rangle, \ \langle \nabla_5 + \alpha\nabla_6 + \nabla_7 \rangle^{O(\alpha)=O(-\alpha)}, \ \langle \nabla_6 \rangle, \ \langle \nabla_6 + \nabla_7 \rangle, \ \langle \nabla_7 \rangle,$$

which gives the following new algebras:

$\mathbf{N}_{361}^{\alpha,\beta}$	: $e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = \alpha e_5$	$e_1e_4 = e_5$	
	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = \beta e_5$		
$\mathbf{N}_{362}^{\alpha,\beta,\gamma,\mu}$	: $e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = \alpha e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4$
	$e_2e_4 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = \mu e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{363}^{\alpha,\beta,\gamma}$	: $e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_3$	
	$e_2e_3 = e_4$	$e_2e_4 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{364}^{\alpha,\beta}$	: $e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_3$	
	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \beta e_5$		
$\mathbf{N}_{365}^{\alpha}$	: $e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_1e_4 = e_5$		
	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{366}^{\alpha,\beta,\gamma}$	: $e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	
	$e_1e_4 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_3e_4 = \gamma e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{367}$	: $e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	
$\mathbf{N}_{368}^{\alpha}$	: $e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$		
	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{369}^{\alpha,\beta}$	: $e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	
	$e_2e_4 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{370}$	: $e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{371}^{\alpha}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$
$\mathbf{N}_{372}^{\alpha}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$		
	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{373}^{\alpha,\beta}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	
	$e_3e_3 = \alpha e_5$	$e_3e_4 = \beta e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{374}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{375}^{\alpha}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$		
	$e_3e_3 = e_5$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{376}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{377}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{378}$	: $e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_4e_4 = e_5$	

4.8. **1-dimensional central extensions of $\mathbf{N}_{08}^4$.** Here we will collect all information about $\mathbf{N}_{08}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{08}^4$	$e_1e_1 = e_2$ $e_1e_3 = e_4$ $e_2e_2 = e_3$ $e_2e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{08}^4) = \Big\langle [\Delta_{ij}] \Big\rangle$ $(i,j) \notin \{(1,1),(1,3),(2,2)\}$	$\phi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t & 0 & 0 & 1 \end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll} \nabla_1 = [\Delta_{12}], & \nabla_2 = [\Delta_{14}], & \nabla_3 = [\Delta_{23}], & \nabla_4 = [\Delta_{24}], \\ \nabla_5 = [\Delta_{33}], & \nabla_6 = [\Delta_{34}], & \nabla_7 = [\Delta_{44}]. \end{array}$$

Take $\theta = \sum\limits_{i=1}^{7} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^4_{08})$. Since

$$\phi^T \begin{pmatrix} 0 & \alpha_1 & 0 & \alpha_2 \\ \alpha_1 & 0 & \alpha_3 & \alpha_4 \\ 0 & \alpha_3 & \alpha_5 & \alpha_6 \\ \alpha_2 & \alpha_4 & \alpha_6 & \alpha_7 \end{pmatrix} \phi = \begin{pmatrix} \alpha^* & \alpha_1^* & \alpha^{**} & \alpha_2^* \\ \alpha_1^* & 0 & \alpha_3^* + \alpha^{**} & \alpha_4^* \\ \alpha^{**} & \alpha_3^* + \alpha^{**} & \alpha_5^* & \alpha_6^* \\ \alpha_2^* & \alpha_4^* & \alpha_6^* & \alpha_7^* \end{pmatrix}$$

we have

$$\begin{array}{llll} \alpha_1^* = \alpha_1 + \alpha_4 t, & \alpha_2^* = \alpha_2 + \alpha_7 t, & \alpha_3^* = \alpha_3 - \alpha_6 t, & \alpha_4^* = \alpha_4, \\ \alpha_5^* = \alpha_5, & \alpha_6^* = \alpha_6, & \alpha_7^* = \alpha_7. \end{array}$$

Since $(\alpha_2, \alpha_4, \alpha_6, \alpha_7) \neq (0,0,0,0)$, we have the following cases:

1. if $\alpha_7 = \alpha_6 = \alpha_4 = 0$, then $\alpha_2 \neq 0$, and we have the family of representatives
$$\langle \alpha\nabla_1 + \nabla_2 + \beta\nabla_3 + \gamma\nabla_5 \rangle;$$
2. if $\alpha_7 = 0, \alpha_6 = 0, \alpha_4 \neq 0$, then by choosing $t = -\alpha_1\alpha_4^{-1}$, we have the family of representatives
$$\langle \alpha\nabla_2 + \beta\nabla_3 + \nabla_4 + \gamma\nabla_5 \rangle;$$
3. if $\alpha_7 = 0, \alpha_6 \neq 0$, then by choosing $t = \alpha_3\alpha_6^{-1}$, we have the family of representatives
$$\langle \alpha\nabla_1 + \beta\nabla_2 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_6 \rangle;$$
4. if $\alpha_7 \neq 0$, then by choosing $t = -\alpha_2\alpha_7^{-1}$, we have the family of representatives
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nu\nabla_6 + \nabla_7 \rangle.$$

Summarizing, we have the following distinct orbits:

$$\langle \alpha\nabla_1 + \nabla_2 + \beta\nabla_3 + \gamma\nabla_5 \rangle, \langle \alpha\nabla_1 + \beta\nabla_2 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_6 \rangle,$$
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nu\nabla_6 + \nabla_7 \rangle, \langle \alpha\nabla_2 + \beta\nabla_3 + \nabla_4 + \gamma\nabla_5 \rangle,$$

which gives the following new algebras:

$\mathbf{N}_{379}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_5$	$e_1e_3 = e_4$	$e_1e_4 = e_5$
		$e_2e_2 = e_3$	$e_2e_3 = e_4 + \beta e_5$	$e_3e_3 = \gamma e_5$	
$\mathbf{N}_{380}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_5$	$e_1e_3 = e_4$	
		$e_1e_4 = \beta e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	
		$e_2e_4 = \gamma e_5$	$e_3e_3 = \mu e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{381}^{\alpha,\beta,\gamma,\mu,\nu}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_5$	$e_1e_3 = e_4$	
		$e_2e_2 = e_3$	$e_2e_3 = e_4 + \beta e_5$	$e_2e_4 = \gamma e_5$	
		$e_3e_3 = \mu e_5$	$e_3e_4 = \nu e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{382}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_3$
		$e_2e_3 = e_4 + \beta e_5$	$e_2e_4 = e_5$	$e_3e_3 = \gamma e_5$	

4.9. **1-dimensional central extensions of $\mathbf{N}^4_{09}$.** Here we will collect all information about $\mathbf{N}^4_{09}$:

		Cohomology	Automorphisms
$\mathbf{N}_{09}^4$	$e_1e_1=e_2$ $e_2e_2=e_3$ $e_3e_3=e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{09}^4)=\Big\langle [\Delta_{ij}]\Big\rangle$ $(i,j)\notin\{(1,1),(2,2),(3,3)\}$	$\phi=\begin{pmatrix} x & 0 & 0 & 0\\ 0 & x^2 & 0 & 0\\ 0 & 0 & x^4 & 0\\ t & 0 & 0 & x^8\end{pmatrix}$

Let us use the following notations:

$$\nabla_1=[\Delta_{12}],\quad \nabla_2=[\Delta_{13}],\quad \nabla_3=[\Delta_{14}],\quad \nabla_4=[\Delta_{23}],$$
$$\nabla_5=[\Delta_{24}],\quad \nabla_6=[\Delta_{34}],\quad \nabla_7=[\Delta_{44}].$$

Take $\theta=\sum\limits_{i=1}^{7}\alpha_i\nabla_i\in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{09}^4)$. Since

$$\phi^T\begin{pmatrix} 0 & \alpha_1 & \alpha_2 & \alpha_3\\ \alpha_1 & 0 & \alpha_4 & \alpha_5\\ \alpha_2 & \alpha_4 & 0 & \alpha_6\\ \alpha_3 & \alpha_5 & \alpha_6 & \alpha_7\end{pmatrix}\phi=\begin{pmatrix} \alpha^* & \alpha_1^* & \alpha_2^* & \alpha_3^*\\ \alpha_1^* & 0 & \alpha_4^* & \alpha_5^*\\ \alpha_2^* & \alpha_4^* & 0 & \alpha_6^*\\ \alpha_3^* & \alpha_5^* & \alpha_6^* & \alpha_7^*\end{pmatrix}$$

we have

$$\alpha_1^*=(\alpha_1x+\alpha_5t)x^2,\quad \alpha_2^*=(\alpha_2x+\alpha_6t)x^4,\quad \alpha_3^*=(\alpha_3x+\alpha_7t)x^8,\quad \alpha_4^*=\alpha_4x^6,$$
$$\alpha_5^*=\alpha_5x^{10},\qquad \alpha_6^*=\alpha_6x^{12},\qquad \alpha_7^*=\alpha_7x^{16}.$$

Since $(\alpha_3,\alpha_5,\alpha_6,\alpha_7)\neq(0,0,0,0)$, we have the following cases:

1. $\alpha_7=\alpha_6=\alpha_5=0$, then $\alpha_3\neq 0$, and we have the following subcases:
   (a) if $\alpha_1=0,\alpha_2=0$, then we have representatives $\langle\nabla_3\rangle$ and $\langle\nabla_3+\nabla_4\rangle$ depending on whether $\alpha_4=0$ or not;
   (b) if $\alpha_1=0,\alpha_2\neq 0$, then by choosing $x=\sqrt[4]{\alpha_2\alpha_3^{-1}},t=0$, we have the family of representatives $\langle\nabla_2+\nabla_3+\alpha\nabla_4\rangle$;
   (c) if $\alpha_1\neq 0$, then by choosing $x=\sqrt[6]{\alpha_1\alpha_3^{-1}},t=0$, we have the family of representatives $\langle\nabla_1+\alpha\nabla_2+\nabla_3+\beta\nabla_4\rangle$.
2. $\alpha_7=0,\alpha_6=0,\alpha_5\neq 0$, then we have the following cases:
   (a) if $\alpha_2=0,\alpha_3=0$, then we have representatives $\langle\nabla_5\rangle$ and $\langle\nabla_4+\nabla_5\rangle$ depending on whether $\alpha_4=0$ or not;
   (b) if $\alpha_2=0,\alpha_3\neq 0$, then choosing $x=\alpha_3\alpha_5^{-1},t=-\alpha_1\alpha_3\alpha_5^{-2}$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_4+\nabla_5\rangle$;
   (c) if $\alpha_2\neq 0$, then by choosing $x=\sqrt[5]{\alpha_2\alpha_5^{-1}},t=-\alpha_1\sqrt[5]{\alpha_2\alpha_5^{-6}}$, we have the family of representatives $\langle\nabla_2+\alpha\nabla_3+\beta\nabla_4+\nabla_5\rangle$.
3. $\alpha_7=0,\alpha_6\neq 0$, then we have the following cases:
   (a) if $\alpha_1\alpha_6=\alpha_2\alpha_5,\alpha_3=0,\alpha_4=0$, then we have representatives $\langle\nabla_6\rangle$ and $\langle\nabla_5+\nabla_6\rangle$ depending on whether $\alpha_5=0$ or not;
   (b) if $\alpha_1\alpha_6=\alpha_2\alpha_5,\alpha_3=0,\alpha_4\neq 0$, then by choosing $x=\sqrt[6]{\alpha_4\alpha_6^{-1}},t=-\alpha_2\sqrt[6]{\alpha_4\alpha_6^{-7}}$, we have the family of representatives $\langle\nabla_4+\alpha\nabla_5+\nabla_6\rangle$;
   (c) if $\alpha_1\alpha_6=\alpha_2\alpha_5,\alpha_3\neq 0$, then by choosing $x=\sqrt[3]{\alpha_3\alpha_6^{-1}},t=-\alpha_2\sqrt[3]{\alpha_3\alpha_6^{-4}}$, we have the family of representatives $\langle\nabla_3+\alpha\nabla_4+\beta\nabla_5+\nabla_6\rangle$;
   (d) if $\alpha_1\alpha_6\neq\alpha_2\alpha_5$, then by choosing

$$x=\sqrt[9]{(\alpha_1\alpha_6-\alpha_2\alpha_5)\alpha_6^{-2}},\ t=-\alpha_2\sqrt[9]{(\alpha_1\alpha_6-\alpha_2\alpha_5)\alpha_6^{-11}},$$

we have the family of representatives

$$\langle\nabla_1+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\nabla_6\rangle.$$

4. $\alpha_7\neq 0$, then we have the following cases:
   (a) if $\alpha_1=0,\alpha_2\alpha_7=\alpha_3\alpha_6,\alpha_4=0,\alpha_5=0$, then choosing $x=1,t=-\frac{\alpha_3}{\alpha_7}$, we have representatives $\langle\nabla_7\rangle$ and $\langle\nabla_6+\nabla_7\rangle$ depending on whether $\alpha_6=0$ or not;
   (b) if $\alpha_1\alpha_7=\alpha_3\alpha_5,\alpha_2\alpha_7=\alpha_3\alpha_6,\alpha_4=0,\alpha_5\neq 0$, then by choosing $x=\sqrt[6]{\alpha_5\alpha_7^{-1}},t=-\alpha_3\sqrt[6]{\alpha_5\alpha_7^{-7}}$, we have the family of representatives $\langle\nabla_5+\alpha\nabla_6+\nabla_7\rangle$;
   (c) if $\alpha_1\alpha_7=\alpha_3\alpha_5,\alpha_2\alpha_7=\alpha_3\alpha_6,\alpha_4\neq 0$, then by choosing
   $$x=\sqrt[10]{\alpha_4\alpha_7^{-1}},\ t=-\alpha_3\sqrt[10]{\alpha_4\alpha_7^{-11}},$$
   we have the family of representatives $\langle\nabla_4+\alpha\nabla_5+\beta\nabla_6+\nabla_7\rangle$;
   (d) $\alpha_1\alpha_7=\alpha_3\alpha_5,\alpha_2\alpha_7\neq\alpha_3\alpha_6$, then by choosing
   $$x=\sqrt[11]{(\alpha_2\alpha_7-\alpha_3\alpha_6)\alpha_7^{-2}},\ t=-\alpha_3\sqrt[11]{(\alpha_2\alpha_7-\alpha_3\alpha_6)\alpha_7^{-13}},$$
   we have the family of representatives
   $$\langle\nabla_2+\alpha\nabla_4+\beta\nabla_5+\gamma\nabla_6+\nabla_7\rangle;$$
   (e) $\alpha_1\alpha_7\neq\alpha_3\alpha_5$, then by choosing
   $$x=\sqrt[13]{(\alpha_1\alpha_7-\alpha_3\alpha_5)\alpha_7^{-2}},\ t=-\alpha_3\sqrt[13]{(\alpha_1\alpha_7-\alpha_3\alpha_5)\alpha_7^{-15}},$$
   we have the family of representatives
   $$\langle\nabla_1+\alpha\nabla_2+\beta\nabla_4+\gamma\nabla_5+\mu\nabla_6+\nabla_7\rangle.$$

Summarizing, we have the following distinct orbits:

$$\langle\nabla_1+\alpha\nabla_2+\nabla_3+\beta\nabla_4\rangle^{O(\alpha,\beta)=O(-\eta_3\alpha,\beta)=O(-\eta_3\alpha,-\beta)=O(\eta_3^2\alpha,-\beta)=O(\eta_3^2\alpha,\beta)=O(\alpha,-\beta)},$$

$$\langle\nabla_1+\alpha\nabla_2+\beta\nabla_4+\gamma\nabla_5+\mu\nabla_6+\nabla_7\rangle^{\begin{array}{l}O(\alpha,\beta,\gamma,\mu)=O(-\eta_{13}^{11}\alpha,\eta_{13}^{10}\beta,\eta_{13}^{6}\gamma,\eta_{13}^{4}\mu)=\\ O(-\eta_{13}^{9}\alpha,-\eta_{13}^{7}\beta,\eta_{13}^{12}\gamma,\eta_{13}^{8}\mu)=O(-\eta_{13}^{7}\alpha,\eta_{13}^{4}\beta,-\eta_{13}^{5}\gamma,\eta_{13}^{12}\mu)=\\ O(-\eta_{13}^{5}\alpha,-\eta_{13}^{1}\beta,-\eta_{13}^{11}\gamma,-\eta_{13}^{3}\mu)=O(-\eta_{13}^{3}\alpha,-\eta_{13}^{11}\beta,\eta_{13}^{4}\gamma,-\eta_{13}^{7}\mu)=\\ O(-\eta_{13}^{1}\alpha,\eta_{13}^{8}\beta,\eta_{13}^{10}\gamma,-\eta_{13}^{11}\mu)=O(\eta_{13}^{12}\alpha,-\eta_{13}^{5}\beta,-\eta_{13}^{3}\gamma,\eta_{13}^{2}\mu)=\\ O(\eta_{13}^{10}\alpha,\eta_{13}^{2}\beta,-\eta_{13}^{9}\gamma,\eta_{13}^{6}\mu)=O(\eta_{13}^{8}\alpha,\eta_{13}^{12}\beta,\eta_{13}^{2}\gamma,\eta_{13}^{10}\mu)=\\ O(\eta_{13}^{6}\alpha,-\eta_{13}^{9}\beta,\eta_{13}^{8}\gamma,-\eta_{13}^{1}\mu)=O(\eta_{13}^{4}\alpha,\eta_{13}^{6}\beta,-\eta_{13}^{1}\gamma,-\eta_{13}^{5}\mu)=\\ O(\eta_{13}^{2}\alpha,-\eta_{13}^{3}\beta,-\eta_{13}^{7}\gamma,-\eta_{13}^{9}\mu)\end{array}},$$

$$\langle\nabla_1+\alpha\nabla_3+\beta\nabla_4+\gamma\nabla_5+\nabla_6\rangle^{\begin{array}{l}O(\alpha,\beta,\gamma,\mu)=O(-\eta_3\alpha,\eta_3^2\beta,\eta_9^2\gamma)=O(\eta_3^2\alpha,-\eta_3\beta,\eta_9^4\gamma)=\\ O(\alpha,\beta,\eta_3^2\gamma)=O(-\eta_3\alpha,\eta_3^2\beta,\eta_9^8\gamma)=O(\eta_3^2\alpha,-\eta_3\beta,-\eta_9\gamma)=\\ O(\alpha,\beta,-\eta_3\gamma)=O(-\eta_3\alpha,\eta_3^2\beta,-\eta_9^5\gamma)=O(\eta_3^2\alpha,-\eta_3\beta,-\eta_9^7\gamma)\end{array}},$$

$$\langle\nabla_2+\nabla_3+\alpha\nabla_4\rangle^{O(\alpha)=O(i\alpha)=O(-\alpha)=(-i\alpha)},$$

$$\langle\nabla_2+\alpha\nabla_3+\beta\nabla_4+\nabla_5\rangle^{O(\alpha,\beta)=O(-\eta_5\alpha,\eta_5^4\beta)=O(\eta_5^2\alpha,-\eta_5^3\beta)=O(-\eta_5^3\alpha,\eta_5^2\beta)=O(\eta_5^4\alpha,-\eta_5\beta)},$$

$$\langle\nabla_2+\alpha\nabla_4+\beta\nabla_5+\gamma\nabla_6+\nabla_7\rangle^{\begin{array}{l}O(\alpha,\beta,\gamma)=O(\eta_{11}^{10}\alpha,\eta_{11}^{6}\beta,\eta_{11}^{4}\gamma=\\ O(-\eta_{11}^{9}\alpha,-\eta_{11}\beta,\eta^{3}8_{11}\gamma)=O(\eta_{11}^{8}\alpha,-\eta_{11}^{7}\beta,-\eta_{11}\gamma)=\\ O(-\eta_{11}^{7}\alpha,\eta_{11}^{2}\beta,-\eta_{11}^{5}\gamma)=O(\eta_{11}^{6}\alpha,\eta_{11}^{8}\beta,-\eta_{11}^{9}\gamma)=\\ O(-\eta_{11}^{5}\alpha,-\eta_{11}^{3}\beta,\eta_{11}^{2}\gamma)=O(\eta_{11}^{4}\alpha,-\eta_{13}^{9}\beta,\eta_{11}^{6}\gamma)=\\ O(-\eta_{11}^{3}\alpha,\eta_{11}^{4}\beta,\eta_{11}^{10}\gamma)=O(\eta_{11}^{2}\alpha,\eta_{11}^{10}\beta,-\eta_{11}^{3}\gamma)=\\ O(-\eta_{11}\alpha,-\eta_{11}^{5}\beta,-\eta_{11}^{7}\gamma)\end{array}},\ \langle\nabla_3\rangle,$$

$$\langle\nabla_3+\nabla_4\rangle,\ \langle\nabla_3+\alpha\nabla_4+\nabla_5\rangle,\ \langle\nabla_3+\alpha\nabla_4+\beta\nabla_5+\nabla_6\rangle^{O(\alpha,\beta)=O(\alpha,-\eta_3\beta)=O(\alpha,\eta_3^2\beta)},$$

$$\langle\nabla_4+\nabla_5\rangle\ \langle\nabla_4+\alpha\nabla_5+\nabla_6\rangle^{\begin{array}{l}O(\alpha)=O(-\eta_3\alpha)=\\ O(\eta_3^2\alpha)\end{array}},$$

$$\langle\nabla_4+\alpha\nabla_5+\beta\nabla_6+\nabla_7\rangle^{\begin{array}{l}O(\alpha,\beta)=O(-\eta_5\alpha,\eta_5^4\beta)=O(\eta_5^2\alpha,-\eta_5^3\beta)=\\ O(-\eta_5^3\alpha,\eta_5^2\beta)=O(\eta_5^4\alpha,-\eta_5\beta)\end{array}},\ \langle\nabla_5\rangle,\ \langle\nabla_5+\nabla_6\rangle,$$

$$\langle\nabla_5+\alpha\nabla_6+\nabla_7\rangle^{O(\alpha)=O(-\eta_3\alpha)=O(\eta_3^2\alpha)},\ \langle\nabla_6\rangle,\ \langle\nabla_6+\nabla_7\rangle,\ \langle\nabla_7\rangle,$$

which gives the following new algebras:

$$\mathbf{N}_{383}^{\alpha,\beta}\ :\ e_1e_1=e_2\quad e_1e_2=e_5\quad e_1e_3=\alpha e_5\quad e_1e_4=e_5$$

		$e_2e_2 = e_3$	$e_2e_3 = \beta e_5$	$e_3e_3 = e_4$		
$\mathbf{N}_{384}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = \alpha e_5$	$e_2e_2 = e_3$	$e_2e_3 = \beta e_5$
		$e_2e_4 = \gamma e_5$	$e_3e_3 = e_4$	$e_3e_4 = \mu e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{385}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_3$	
		$e_2e_3 = \beta e_5$	$e_2e_4 = \gamma e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{386}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_1e_4 = e_5$		
		$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$	$e_3e_3 = e_4$		
$\mathbf{N}_{387}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_3$	
		$e_2e_3 = \beta e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$		
$\mathbf{N}_{388}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$	
		$e_2e_4 = \beta e_5$	$e_3e_3 = e_4$	$e_3e_4 = \gamma e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{389}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$	$e_3e_3 = e_4$	
$\mathbf{N}_{390}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{391}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$		
		$e_2e_3 = \alpha e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$		
$\mathbf{N}_{392}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$	
		$e_2e_4 = \beta e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{393}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{394}^{\alpha}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_5$		
		$e_2e_4 = \alpha e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{395}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_5$	$e_2e_4 = \alpha e_5$	
		$e_3e_3 = e_4$	$e_3e_4 = \beta e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{396}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_4 = e_5$	$e_3e_3 = e_4$	
$\mathbf{N}_{397}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_4 = e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{398}^{\alpha}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_4 = e_5$		
		$e_3e_3 = e_4$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{399}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{400}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{401}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_3e_3 = e_4$	$e_4e_4 = e_5$	

4.10. **1-dimensional central extensions of $\mathbf{N}_{10}^4$.** Here we will collect all information about $\mathbf{N}_{10}^4$:

		Cohomology	Automorphisms
$\mathbf{N}_{10}^4$	$e_1e_1 = e_2$ $e_1e_2 = e_4$ $e_2e_2 = e_3$ $e_3e_3 = e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{10}^4) = \Big\langle [\Delta_{ij}] \Big\rangle$ $(i,j) \notin \{(1,1),(1,2),(2,2)\}$	$\phi_k = \begin{pmatrix} \eta^k & 0 & 0 & 0 \\ 0 & \eta^{2k} & 0 & 0 \\ 0 & 0 & \eta^{4k} & 0 \\ t & 0 & 0 & \eta^{8k} \end{pmatrix}$ $\eta = -\eta_5,\ k = 0,1,2,3,4$

Let us use the following notations:

$$\nabla_1 = [\Delta_{13}], \quad \nabla_2 = [\Delta_{14}], \quad \nabla_3 = [\Delta_{23}], \quad \nabla_4 = [\Delta_{24}],$$
$$\nabla_5 = [\Delta_{33}], \quad \nabla_6 = [\Delta_{34}], \quad \nabla_7 = [\Delta_{44}].$$

Take $\theta = \sum\limits_{i=1}^{7} \alpha_i \nabla_i \in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}^4_{10})$. Since

$$\phi_k^T \begin{pmatrix} 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & 0 & \alpha_3 & \alpha_4 \\ \alpha_1 & \alpha_3 & \alpha_5 & \alpha_6 \\ \alpha_2 & \alpha_4 & \alpha_6 & \alpha_7 \end{pmatrix} \phi_k = \begin{pmatrix} \alpha^* & \alpha^{**} & \alpha_1^* & \alpha_2^* \\ \alpha^{**} & 0 & \alpha_3^* & \alpha_4^* \\ \alpha_1^* & \alpha_3^* & \alpha_5^*+\alpha^{**} & \alpha_6^* \\ \alpha_2^* & \alpha_4^* & \alpha_6^* & \alpha_7^* \end{pmatrix},$$

we have

$$\begin{array}{llll} \alpha_1^* = \eta^{4k}(\eta^k\alpha_1 + t\alpha_6), & \alpha_2^* = \eta^{8k}(\eta^k\alpha_2 + t\alpha_7), & \alpha_3^* = \eta^{6k}\alpha_3, & \alpha_4^* = \eta^{10k}\alpha_4, \\ \alpha_5^* = -t\eta^{2k}\alpha_4 + \eta^{8k}\alpha_5, & \alpha_6^* = \eta^{12k}\alpha_6, & \alpha_7^* = \eta^{16k}\alpha_7. & \end{array}$$

Since $(\alpha_2, \alpha_4, \alpha_6, \alpha_7) \neq (0,0,0,0)$, we have the following cases:

1. if $\alpha_7 = 0, \alpha_6 = 0, \alpha_4 = 0$, then $\alpha_2 \neq 0$, and we have the family of representatives
$$\langle \alpha\nabla_1 + \nabla_2 + \beta\nabla_3 + \gamma\nabla_5 \rangle;$$
2. if $\alpha_7 = 0, \alpha_6 = 0, \alpha_4 \neq 0$, then we have the family of representatives
$$\langle \alpha\nabla_1 + \beta\nabla_2 + \gamma\nabla_3 + \nabla_4 \rangle;$$
3. if $\alpha_7 = 0, \alpha_6 \neq 0$, then we have the family of representatives
$$\langle \alpha\nabla_2 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_6 \rangle;$$
4. if $\alpha_7 \neq 0$, then we have the family of representatives
$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nu\nabla_6 + \nabla_7 \rangle.$$

Summarizing, we have the following distinct orbits:

$$\langle \alpha\nabla_1 + \beta\nabla_2 + \gamma\nabla_3 + \nabla_4 \rangle_{\substack{O(\alpha,\beta,\gamma) = O(\alpha,\eta_5^4\beta,-\eta_5\gamma) = O(\alpha,-\eta_5^3\beta,\eta_5^2\gamma) = \\ O(\alpha,\eta_5^2\beta,-\eta_5^3\gamma) = O(\alpha,-\eta_5\beta,\eta_5^4\gamma)}},$$

$$\langle \alpha\nabla_1 + \nabla_2 + \beta\nabla_3 + \gamma\nabla_5 \rangle_{\substack{O(\alpha,\beta,\gamma) = O(-\eta_5\alpha,\eta_5^2\beta,\eta_5^4\gamma) = O(\eta_5^2\alpha,\eta_5^4\beta,-\eta_5^3\gamma) = \\ O(-\eta_5^3\alpha,-\eta_5\beta,\eta_5^2\gamma) = O(\eta_5^4\alpha,-\eta_5^3\beta,-\eta_5\gamma)}},$$

$$\langle \alpha\nabla_1 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nu\nabla_6 + \nabla_7 \rangle_{\substack{O(\alpha,\beta,\gamma,\mu,\nu) = O(\eta_5^4\alpha,\beta,\eta_5^4\gamma,\eta_5^2\mu,-\eta_5\nu) = \\ O(-\eta_5^3\alpha,\beta,-\eta_5^3\gamma,\eta_5^4\mu,\eta_5^2\nu) = O(\eta_5^2\alpha,\beta,\eta_5^2\gamma,-\eta_5\mu,-\eta_5^3\nu) = \\ O(-\eta_5\alpha,\beta,-\eta_5\gamma,-\eta_5^3\mu,\eta_5^4\nu)}},$$

$$\langle \alpha\nabla_2 + \beta\nabla_3 + \gamma\nabla_4 + \mu\nabla_5 + \nabla_6 \rangle_{\substack{O(\alpha,\beta,\gamma,\mu) = O(\eta_5^2\alpha,\eta_5^4\beta,-\eta_5^3\gamma,-\eta_5\mu) = O(\eta_5^4\alpha,-\eta_5^3\beta,-\eta_5\gamma,\eta_5^2\mu) = \\ O(-\eta_5\alpha,\eta_5^2\beta,\eta_5^4\gamma,-\eta_5^3\mu) = O(-\eta_5^3\alpha,-\eta_5\beta,\eta_5^2\gamma,\eta_5^4\mu)}},$$

which gives the following new algebras:

$$\begin{array}{lllll}
\mathbf{N}_{402}^{\alpha,\beta,\gamma} & : & e_1e_1 = e_2 & e_1e_2 = e_4 & e_1e_3 = \alpha e_5 & e_1e_4 = \beta e_5 \\
& & e_2e_2 = e_3 & e_2e_3 = \gamma e_5 & e_2e_4 = e_5 & e_3e_3 = e_4 \\
\mathbf{N}_{403}^{\alpha,\beta,\gamma} & : & e_1e_1 = e_2 & e_1e_2 = e_4 & e_1e_3 = \alpha e_5 & e_1e_4 = e_5 \\
& & e_2e_2 = e_3 & e_2e_3 = \beta e_5 & e_3e_3 = e_4 + \gamma e_5 & \\
\mathbf{N}_{404}^{\alpha,\beta,\gamma,\mu,\nu} & : & e_1e_1 = e_2 & e_1e_2 = e_4 & e_1e_3 = \alpha e_5 & \\
& & e_2e_2 = e_3 & e_2e_3 = \beta e_5 & e_2e_4 = \gamma e_5 & \\
& & e_3e_3 = e_4 + \mu e_5 & e_3e_4 = \nu e_5 & e_4e_4 = e_5 & \\
\mathbf{N}_{405}^{\alpha,\beta,\gamma,\mu} & : & e_1e_1 = e_2 & e_1e_2 = e_4 & e_1e_4 = \alpha e_5 & e_2e_2 = e_3 \\
& & e_2e_3 = \beta e_5 & e_2e_4 = \gamma e_5 & e_3e_3 = e_4 + \mu e_5 & e_3e_4 = e_5
\end{array}$$

4.11. **1-dimensional central extensions of $\mathbf{N}^4_{11}(\lambda)$.** Here we will collect all information about $\mathbf{N}^4_{11}(\lambda)$:

		Cohomology	Automorphisms
$\mathbf{N}_{11}^{4}(\lambda)$	$e_1e_1=e_2$ $e_1e_2=\lambda e_4$ $e_2e_2=e_3$ $e_2e_3=e_4$ $e_3e_3=e_4$	$\mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{11}^{4}(\lambda))=\Big\langle[\Delta_{ij}]\Big\rangle$ $(i,j)\notin\{(1,1),(2,2),(3,3)\}$	$\phi=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\t&0&0&1\end{pmatrix}$

Let us use the following notations:

$$\begin{array}{llll}\nabla_1=[\Delta_{12}], & \nabla_2=[\Delta_{13}], & \nabla_3=[\Delta_{14}], & \nabla_4=[\Delta_{23}],\\ \nabla_5=[\Delta_{24}], & \nabla_6=[\Delta_{34}], & \nabla_7=[\Delta_{44}]. &\end{array}$$

Take $\theta=\sum\limits_{i=1}^{7}\alpha_i\nabla_i\in \mathrm{H}^2_{\mathfrak{C}}(\mathbf{N}_{11}^{4}(\lambda))$. Since

$$\phi^T\begin{pmatrix}0&\alpha_1&\alpha_2&\alpha_3\\ \alpha_1&0&\alpha_4&\alpha_5\\ \alpha_2&\alpha_4&0&\alpha_6\\ \alpha_3&\alpha_5&\alpha_6&\alpha_7\end{pmatrix}\phi=\begin{pmatrix}\alpha^*&\alpha_1^*&\alpha_2^*&\alpha_3^*\\ \alpha_1^*&0&\alpha_4^*&\alpha_5^*\\ \alpha_2^*&\alpha_4^*&0&\alpha_6^*\\ \alpha_3^*&\alpha_5^*&\alpha_6^*&\alpha_7^*\end{pmatrix}$$

we have

$$\begin{array}{llll}\alpha_1^*=\alpha_1+\alpha_5t, & \alpha_2^*=\alpha_2+\alpha_6t, & \alpha_3^*=\alpha_3+\alpha_7t, & \alpha_4^*=\alpha_4,\\ \alpha_5^*=\alpha_5, & \alpha_6^*=\alpha_6, & \alpha_7^*=\alpha_7. &\end{array}$$

Since $(\alpha_3,\alpha_5,\alpha_6,\alpha_7)\neq(0,0,0,0)$, we have the following cases:

1. if $\alpha_7=0,\alpha_6=0,\alpha_5=0$, then $\alpha_3\neq0$ and we have the family of representatives
$$\langle\alpha\nabla_1+\beta\nabla_2+\nabla_3+\gamma\nabla_4\rangle;$$
2. if $\alpha_7=0,\alpha_6=0,\alpha_5\neq0$ then by choosing $t=-\alpha_1\alpha_5^{-1}$, we have the family of representatives
$$\langle\alpha\nabla_2+\beta\nabla_3+\gamma\nabla_4+\nabla_5\rangle;$$
3. if $\alpha_7=0,\alpha_6\neq0$ then by choosing $t=-\alpha_2\alpha_6^{-1}$, we have the family of representatives
$$\langle\alpha\nabla_1+\beta\nabla_3+\gamma\nabla_4+\mu\nabla_5+\nabla_6\rangle;$$
4. if $\alpha_7\neq0$ then by choosing $t=-\alpha_3\alpha_7^{-1}$, we have the family of representatives
$$\langle\alpha\nabla_1+\beta\nabla_2+\gamma\nabla_4+\mu\nabla_5+\nu\nabla_6+\nabla_7\rangle.$$

Summarizing, we have the following distinct orbits:

$$\langle\alpha\nabla_1+\beta\nabla_2+\nabla_3+\gamma\nabla_4\rangle,\ \langle\alpha\nabla_1+\beta\nabla_2+\gamma\nabla_4+\mu\nabla_5+\nu\nabla_6+\nabla_7\rangle,$$
$$\langle\alpha\nabla_1+\beta\nabla_3+\gamma\nabla_4+\mu\nabla_5+\nabla_6\rangle,\ \langle\alpha\nabla_2+\beta\nabla_3+\gamma\nabla_4+\nabla_5\rangle,$$

which gives the following new algebras:

$$\begin{array}{lllll}
\mathbf{N}_{406}^{\lambda,\alpha,\beta,\gamma} & : & e_1e_1=e_2 & e_1e_2=\lambda e_4+\alpha e_5 & e_1e_3=\beta e_5 & e_1e_4=e_5\\
 & & e_2e_2=e_3 & e_2e_3=e_4+\gamma e_5 & e_3e_3=e_4 & \\
\mathbf{N}_{407}^{\lambda,\alpha,\beta,\gamma,\mu,\nu} & : & e_1e_1=e_2 & e_1e_2=\lambda e_4+\alpha e_5 & e_1e_3=\beta e_5 & \\
 & & e_2e_2=e_3 & e_2e_3=e_4+\gamma e_5 & e_2e_4=\mu e_5 & \\
 & & e_3e_3=e_4 & e_3e_4=\nu e_5 & e_4e_4=e_5 & \\
\mathbf{N}_{408}^{\lambda,\alpha,\beta,\gamma} & : & e_1e_1=e_2 & e_1e_2=\lambda e_4+\alpha e_5 & e_1e_4=\beta e_5 & e_2e_2=e_3\\
 & & e_2e_3=e_4+\gamma e_5 & e_2e_4=\mu e_5 & e_3e_3=e_4 & e_3e_4=e_5
\end{array}$$

$\mathbf{N}_{409}^{\lambda,\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = \lambda e_4$	$e_1e_3 = \alpha e_5$	$e_1e_4 = \beta e_5$
		$e_2e_2 = e_3$	$e_2e_3 = e_4 + \gamma e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$

Remark 2. Note that the algebras $\mathbf{N}_{11}^4(\lambda)$ and $\mathbf{N}_{11}^4(-\lambda)$ are isomorphic. Hence, there are some additional isomorphism relations for algebras from the present subsection $\mathbf{N}^{\lambda,\Xi} \cong \mathbf{N}^{-\lambda,\Xi}$.

5. Classification theorem.

Theorem 5.1. *Let* $\mathbf{N}$ *be a complex* 5*-dimensional nilpotent commutative algebra. Then we have one of the following situations.*

1. *If* $\mathbf{N}$ *is associative, then* $\mathbf{N}$ *is isomorphic to one algebra listed in* [15].
2. *If* $\mathbf{N}$ *is a non-associative Jordan algebra, then* $\mathbf{N}$ *is isomorphic to one algebra listed in* [9].
3. *If* $\mathbf{N}$ *is a non-Jordan* $\mathfrak{CD}$*-algebra, then* $\mathbf{N}$ *is isomorphic to one algebra listed in* [11].
4. *If* $\mathbf{N}$ *is a non-*$\mathfrak{CD}$*-algebra, then* $\mathbf{N}$ *is isomorphic to one algebra listed in the following list.*

$\mathbf{N}_{01}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_4$	
$\mathbf{N}_{02}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$
$\mathbf{N}_{03}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_3e_3 = e_4$	
$\mathbf{N}_{04}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_4$	$e_3e_3 = e_4$
$\mathbf{N}_{05}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
$\mathbf{N}_{06}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_1e_3 = e_4$	$e_2e_2 = e_3$
$\mathbf{N}_{07}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	
$\mathbf{N}_{08}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_2e_3 = e_4$
$\mathbf{N}_{09}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_3e_3 = e_4$	
$\mathbf{N}_{10}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_1e_2 = e_4$	$e_3e_3 = e_4$
$\mathbf{N}_{11}^{\lambda}$	:	$e_1e_1 = e_2$	$e_1e_2 = \lambda e_4$	$e_2e_2 = e_3$	
		$e_2e_3 = e_4$	$e_3e_3 = e_4$		
$\mathbf{N}_{12}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{13}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_4$	
		$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	
$\mathbf{N}_{14}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_3 = e_4$	$e_3e_3 = e_5$		
$\mathbf{N}_{15}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{16}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_1e_3 = e_5$	
		$e_2e_2 = e_3$	$e_2e_3 = e_5$		
$\mathbf{N}_{17}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_2e_2 = e_3$	$e_2e_3 = e_5$
$\mathbf{N}_{18}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_2e_2 = e_3$	
		$e_2e_3 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{19}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_2e_2 = e_3$	$e_3e_3 = e_5$
$\mathbf{N}_{20}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4 + \alpha e_5$		$e_1e_3 = e_4$
		$e_2e_2 = e_3$	$e_2e_3 = e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{21}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_1e_3 = e_4$	
		$e_2e_2 = e_3$	$e_2e_3 = e_5$		
$\mathbf{N}_{22}^{\alpha\neq 1}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_1e_3 = e_4$	
		$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{23}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = \beta e_4 + \alpha e_5$		$e_1e_3 = e_4$
		$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	
$\mathbf{N}_{24}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_4 + e_5$		$e_2e_2 = e_3$

		$e_2e_3 = e_4$	$e_3e_3 = e_5$		
$\mathbf{N}_{25}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_2e_3 = e_4$	$e_3e_3 = e_5$		
$\mathbf{N}_{26}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_4$	$e_1e_3 = e_5$	
		$e_2e_2 = e_3$	$e_3e_3 = e_4$		
$\mathbf{N}_{27}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_2e_3 = e_5$
$\mathbf{N}_{28}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_2e_3 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{29}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_3e_3 = e_5$
$\mathbf{N}_{30}$	:	$e_1e_1 = e_2$	$e_2e_2 = e_3$	$e_2e_3 = e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{31}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_4 = e_5$	
		$e_2e_2 = e_5$	$e_2e_3 = e_5$		
$\mathbf{N}_{32}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_4 = e_5$	
		$e_2e_2 = \alpha e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{33}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_4 = e_5$	
		$e_2e_2 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{34}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_2e_3 = e_5$
$\mathbf{N}_{35}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{36}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_5$	
		$e_2e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{37}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_5$	
		$e_2e_4 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{38}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = \alpha e_5$	
		$e_2e_4 = e_5$	$e_3e_3 = e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{39}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_5$	
		$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{40}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_2 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{41}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_5$	
		$e_2e_4 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{42}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{43}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{44}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_4 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{45}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_3e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{46}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_3e_4 = e_5$	
$\mathbf{N}_{47}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = e_5$	
		$e_2e_2 = e_3$	$e_4e_4 = e_5$		
$\mathbf{N}_{48}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_4 = e_5$	
		$e_2e_2 = e_3$	$e_3e_3 = e_5$		
$\mathbf{N}_{49}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_5$	$e_1e_4 = e_5$	
		$e_2e_2 = e_3$	$e_2e_3 = e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{50}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_5$	$e_2e_2 = e_3$	$e_2e_3 = \beta e_5$
		$e_2e_4 = e_5$	$e_3e_3 = e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{51}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_5$	$e_2e_2 = e_3$	
		$e_2e_3 = e_5$	$e_3e_3 = e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{52}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_2 = e_3$	
		$e_2e_3 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{53}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = \alpha e_5$	$e_2e_2 = e_3$	
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{54}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_2 = e_3$	
		$e_2e_4 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{55}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_2 = e_3$	
		$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{56}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_2 = e_3$	$e_3e_4 = e_5$

$\mathbf{N}_{57}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=e_5$	
		$e_2e_2=e_3$	$e_2e_3=e_5$		
$\mathbf{N}_{58}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	
		$e_2e_3=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{59}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_2e_4=e_5$
$\mathbf{N}_{60}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	
		$e_2e_4=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{61}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_4e_4=e_5$
$\mathbf{N}_{62}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=e_5$
$\mathbf{N}_{63}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$	
		$e_2e_3=e_5$	$e_2e_4=e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{64}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	
		$e_2e_4=e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{65}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_3e_3=e_5$
$\mathbf{N}_{66}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_3e_4=e_5$
$\mathbf{N}_{67}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{68}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_5$
$\mathbf{N}_{69}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_4=e_5$
$\mathbf{N}_{70}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{71}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_3e_4=e_5$	
$\mathbf{N}_{72}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	
		$e_1e_4=\frac{3}{4}e_5$	$e_2e_2=e_4$	$e_2e_3=-\frac{3}{4}e_5$	
$\mathbf{N}_{73}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$
		$e_2e_2=e_4$	$e_2e_3=3e_5$	$e_2e_4=\beta e_5$	$e_4e_4=e_5$
$\mathbf{N}_{74}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	
		$e_1e_4=\alpha e_5$	$e_2e_2=e_4$	$e_2e_4=e_5$	
$\mathbf{N}_{75}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	
		$e_2e_2=e_4$	$e_2e_4=\alpha e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{76}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=(1+\alpha)e_5$	
		$e_2e_2=e_4$	$e_2e_3=3\alpha e_5$		
$\mathbf{N}_{77}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=e_4$
		$e_2e_3=3e_5$	$e_2e_4=\alpha e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{78}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=e_4$
		$e_2e_3=3\alpha e_5$	$e_2e_4=\beta e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{79}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_2=e_4$	$e_2e_4=e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{80}^{\alpha\neq 1}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_4$	
		$e_2e_4=\alpha e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{81}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_4$	
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{82}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{83}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_5$	$e_2e_2=\alpha e_5$
		$e_2e_3=e_4$	$e_2e_4=\beta e_5$	$e_3e_3=\gamma e_5$	$e_4e_4=e_5$
$\mathbf{N}_{84}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=e_5$	$e_2e_3=e_4$
$\mathbf{N}_{85}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=e_5$	$e_2e_2=\alpha e_5$
		$e_2e_3=e_4$	$e_2e_4=\beta e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{86}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=e_5$	
		$e_2e_2=e_5$	$e_2e_3=e_4$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{87}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=e_5$	
		$e_2e_3=e_4$	$e_3e_3=e_5$		
$\mathbf{N}_{88}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_2e_2=e_5$	$e_2e_3=e_4$
		$e_2e_4=\alpha e_5$	$e_3e_3=\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{89}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_2e_2=e_5$	

	$e_2e_3 = e_4$	$e_2e_4 = \alpha e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{90}^{\alpha}$:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{91}$:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{92}$:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	
	$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{93}$:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{94}$:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_2e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{95}^{\alpha}$:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_2 = e_5$	
	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{96}^{\alpha,\beta}$:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_2 = e_5$	$e_2e_3 = e_4$
	$e_2e_4 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{97}$:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$
$\mathbf{N}_{98}$:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{99}^{\alpha}$:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{100}$:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	
	$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{101}$:	$e_1e_1 = e_2$	$e_1e_3 = e_5$	$e_2e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{102}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	
$\mathbf{N}_{103}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_5$	$e_2e_3 = e_4$
$\mathbf{N}_{104}^{\alpha}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_5$	
	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{105}^{\alpha}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = \alpha e_5$	
	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{106}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_2 = e_5$	
	$e_2e_3 = e_4$	$e_3e_3 = e_5$		
$\mathbf{N}_{107}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$
$\mathbf{N}_{108}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{109}$:	$e_1e_1 = e_2$	$e_1e_4 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = e_5$
$\mathbf{N}_{110}$:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_2e_4 = e_5$
$\mathbf{N}_{111}$:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{112}^{\alpha}$:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = \alpha e_5$	$e_3e_3 = e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{113}$:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{114}$:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	
	$e_2e_4 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{115}$:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{116}$:	$e_1e_1 = e_2$	$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{117}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	
$\mathbf{N}_{118}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{119}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	
	$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{120}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{121}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{122}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_3e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{123}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{124}$:	$e_1e_1 = e_2$	$e_2e_3 = e_4$	$e_4e_4 = e_5$	

$\mathbf{N}_{125}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_5$	
		$e_2e_2=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{126}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=\alpha e_5$	$e_2e_2=e_5$
		$e_3e_3=e_4+\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{127}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=\alpha e_5$	
		$e_2e_3=e_5$	$e_3e_3=e_4$	$e_4e_4=e_5$	
$\mathbf{N}_{128}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=e_5$	
		$e_3e_3=e_4$	$e_4e_4=e_5$		
$\mathbf{N}_{129}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_2=e_5$	
		$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{130}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_2=e_5$	
		$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{131}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_2e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{132}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_3e_3=e_4+e_5$	$e_4e_4=e_5$
$\mathbf{N}_{133}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{134}$	:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_3e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{135}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_4$	$e_1e_4=e_5$
		$e_2e_2=e_4$	$e_2e_3=-2e_5$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{136}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_4$	
		$e_2e_2=e_4+\alpha e_5$	$e_2e_3=\beta e_5$		
		$e_3e_3=\gamma e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{137}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=e_4$	$e_2e_2=e_4$
		$e_2e_3=\alpha e_5$	$e_2e_4=\beta e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{138}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=\alpha e_5$	
		$e_2e_2=e_4+e_5$	$e_2e_4=e_5$	$e_3e_3=4e_5$	
$\mathbf{N}_{139}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=e_5$	
		$e_2e_2=e_4$	$e_2e_3=\alpha e_5$		
$\mathbf{N}_{140}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=e_5$	
		$e_2e_2=e_4$	$e_2e_3=\alpha e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{141}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=e_5$	
		$e_2e_2=e_4$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{142}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4+e_5$	
		$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{143}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	
		$e_2e_3=e_5$	$e_2e_4=\alpha e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{144}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	
		$e_2e_3=e_5$	$e_3e_3=\alpha e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{145}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$		
$\mathbf{N}_{146}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	
		$e_2e_4=e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{147}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	
		$e_3e_3=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{148}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{149}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{150}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=\alpha e_5$	$e_2e_2=e_4$
		$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=e_4+\mu e_5$	$e_4e_4=e_5$
$\mathbf{N}_{151}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=e_5$	
		$e_2e_2=e_4$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4+\beta e_5$	
$\mathbf{N}_{152}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_4$
		$e_2e_3=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{153}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_4$
		$e_2e_4=e_5$	$e_3e_3=e_4+\beta e_5$		$e_3e_4=\gamma e_5$

$\mathbf{N}_{154}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_4$	$e_2e_3=\alpha e_5$
		$e_2e_4=\beta e_5$	$e_3e_3=e_4+\gamma e_5$		$e_4e_4=e_5$
$\mathbf{N}_{155}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	$e_3e_3=e_4$
$\mathbf{N}_{156}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	$e_2e_3=e_5$
		$e_3e_3=e_4+\alpha e_5$			
$\mathbf{N}_{157}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	
		$e_2e_3=\alpha e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{158}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	$e_2e_4=e_5$
		$e_3e_3=e_4+\alpha e_5$	$e_3e_4=\beta e_5$		
$\mathbf{N}_{159}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_4$	
		$e_3e_3=e_4+e_5$			
$\mathbf{N}_{160}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_3=e_5$	$e_2e_4=\alpha e_5$
		$e_3e_3=e_4+\beta e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{161}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_3=e_5$	
		$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{162}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_4=e_5$	
		$e_3e_3=e_4+e_5$	$e_3e_4=\alpha e_5$		
$\mathbf{N}_{163}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_4=e_5$	
		$e_3e_3=e_4+\alpha e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{164}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_2e_4=e_5$	
		$e_3e_3=e_4$	$e_3e_4=\alpha e_5$		
$\mathbf{N}_{165}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_3e_3=e_4+e_5$	$e_4e_4=e_5$
$\mathbf{N}_{166}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{167}$	:	$e_1e_1=e_2$	$e_2e_2=e_4$	$e_3e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{168}^{\lambda\neq 1;2}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_1e_4=(\lambda-4)e_5$	$e_2e_2=\lambda e_4+4(1-\lambda)(\lambda-2)e_5$		
		$e_2e_3=-\lambda(\lambda+2)e_5$			
$\mathbf{N}_{169}^{\alpha\neq 0}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$
		$e_2e_2=\alpha e_5$	$e_2e_3=-2e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{170}^{\lambda,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_1e_4=\alpha e_5$	$e_2e_2=\lambda e_4$	$e_2e_3=(1+\alpha(3\lambda-2))e_5$	
$\mathbf{N}_{171}^{\lambda}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_1e_4=e_5$	$e_2e_2=\lambda e_4$		
		$e_2e_3=(3\lambda-2)e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{172}^{\lambda\neq 0,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4+e_5$	$e_2e_3=\alpha e_5$	$e_2e_4=\frac{\lambda}{4}e_5$	$e_3e_3=e_5$
$\mathbf{N}_{173}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=e_5$
		$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{174}^{\lambda\neq -2,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4+\alpha e_5$	$e_2e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{175}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=-2e_4+\alpha e_5$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$	$e_4e_4=e_5$
$\mathbf{N}_{176}^{\lambda,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4+\alpha e_5$	$e_3e_3=e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{177}^{\lambda}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4+e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{178}^{\lambda}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_2e_2=\lambda e_4$	$e_2e_3=e_5$	$e_2e_4=e_5$	
$\mathbf{N}_{179}^{\lambda,\alpha\neq 0}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=\lambda e_4$
		$e_2e_3=e_5$	$e_2e_4=\alpha e_5$	$e_3e_3=e_5$	
$\mathbf{N}_{180}^{\lambda,\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=\lambda e_4$
		$e_2e_3=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{181}^{\alpha\neq 0}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=-2e_4$

		$e_2e_3 = \alpha e_5$	$e_3e_3 = e_5$	$e_3e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{182}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = -2e_4$	$e_2e_3 = e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{183}^{\lambda\neq 2}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_2e_4 = e_5$		
$\mathbf{N}_{184}^{\lambda,\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_2e_4 = \alpha e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{185}^{\lambda}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_3e_3 = e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{186}^{\lambda}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{187}^{\lambda}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = \lambda e_4$	$e_4e_4 = e_5$		
$\mathbf{N}_{188}^{\alpha,\beta}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_5$
		$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{189}^{\alpha}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = e_5$	$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{190}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = -e_5$	
$\mathbf{N}_{191}^{\alpha}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_3 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{191}^{\alpha}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{192}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_3e_3 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{193}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{194}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{195}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{196}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
$\mathbf{N}_{197}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_2 = e_5$
$\mathbf{N}_{198}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{199}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_5$	$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{200}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_3 = e_5$
$\mathbf{N}_{201}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = -e_5$	
$\mathbf{N}_{202}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{203}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_3e_3 = e_5$
$\mathbf{N}_{204}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_3e_3 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{205}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{206}^{\alpha,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_5$	
		$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{207}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_5$	
		$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{208}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_5$	
		$e_2e_4 = e_5$	$e_3e_3 = -e_5$		
$\mathbf{N}_{209}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_5$	
		$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{210}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$

$\mathbf{N}_{211}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_3e_3 = e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{212}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_3e_3 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{213}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{214}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_4e_4 = e_5$	
$\mathbf{N}_{215}^{\alpha,\beta}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$
		$e_2e_2 = e_4$	$e_3e_3 = \beta e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{216}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = e_4 + \alpha e_5$	$e_2e_3 = \beta e_5$		
		$e_3e_3 = \gamma e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{217}^{\alpha}$	:	$e_1e_1 = \alpha e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = -e_5$	
$\mathbf{N}_{218}^{\alpha}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	
		$e_2e_2 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{219}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = 2e_5$	
		$e_2e_2 = e_4 + e_5$	$e_2e_3 = e_5$		
$\mathbf{N}_{220}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_4 + \alpha e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{221}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_4$	$e_2e_3 = \alpha e_5$		
$\mathbf{N}_{222}^{\alpha\neq 0}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_4$
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = -e_5$	
$\mathbf{N}_{223}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{224}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_4$	$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{225}^{\alpha,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4 + e_5$	
		$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{226}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_2e_3 = e_5$	$e_3e_3 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{227}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$		
$\mathbf{N}_{228}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_3e_3 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{229}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	
		$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{230}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{231}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{232}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4 + \beta e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = \mu e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{233}^{\alpha,\beta}$	:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4$	$e_3e_3 = \beta e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{234}^{\alpha,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4 + e_5$	$e_3e_3 = \beta e_5$		
$\mathbf{N}_{235}^{\alpha,\beta,\gamma}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_2 = \alpha e_5$
		$e_2e_3 = e_4$	$e_2e_4 = \beta e_5$	$e_3e_3 = \gamma e_5$	$e_3e_4 = e_5$
$\mathbf{N}_{236}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_5$	$e_2e_3 = e_4$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{237}^{\alpha\neq 0,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	
		$e_2e_3 = e_4 + e_5$	$e_2e_4 = e_5$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{238}^{\alpha\neq 0,\beta}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = \alpha e_5$	
		$e_2e_3 = e_4$	$e_2e_4 = e_5$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{239}^{\alpha}$	:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_3 = e_4$	$e_3e_3 = \alpha e_5$		

$\mathbf{N}_{240}^{\alpha,\beta}$:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4 + e_5$	
	$e_3e_3 = \alpha e_5$	$e_3e_4 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{241}$:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	
	$e_3e_3 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{242}^{\alpha}$:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	
	$e_3e_3 = e_5$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{243}$:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{244}$:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	
	$e_3e_4 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{245}$:	$e_1e_2 = e_3$	$e_1e_3 = e_4$	$e_2e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{246}^{\alpha,\beta,\gamma}$:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	$e_2e_2 = e_5$
	$e_2e_3 = \beta e_5$	$e_3e_3 = e_4$	$e_3e_4 = \gamma e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{247}^{\alpha,\beta}$:	$e_1e_1 = e_5$	$e_1e_2 = e_3$	$e_1e_4 = \alpha e_5$	$e_2e_2 = e_5$
	$e_2e_4 = \beta e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{248}$:	$e_1e_2 = e_3$	$e_1e_3 = e_5$	$e_1e_4 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{249}^{\alpha}$:	$e_1e_2 = e_3$	$e_1e_3 = e_5$	$e_1e_4 = e_5$	
	$e_2e_2 = e_5$	$e_2e_3 = \alpha e_5$	$e_3e_3 = e_4$	
$\mathbf{N}_{250}$:	$e_1e_2 = e_3$	$e_1e_3 = e_5$	$e_1e_4 = e_5$	
	$e_2e_3 = e_5$	$e_3e_3 = e_4$		
$\mathbf{N}_{251}^{\alpha}$:	$e_1e_2 = e_3$	$e_1e_3 = e_5$	$e_1e_4 = e_5$	
	$e_2e_3 = \alpha e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$	
$\mathbf{N}_{252}^{\alpha,\beta}$:	$e_1e_2 = e_3$	$e_1e_3 = e_5$	$e_2e_2 = e_5$	$e_2e_3 = \alpha e_5$
	$e_3e_3 = e_4$	$e_3e_4 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{253}^{\alpha}$:	$e_1e_2 = e_3$	$e_1e_3 = e_5$	$e_2e_3 = e_5$	
	$e_3e_3 = e_4$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{254}$:	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_3e_3 = e_4$	
$\mathbf{N}_{255}^{\alpha}$:	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_2e_2 = e_5$	
	$e_2e_3 = \alpha e_5$	$e_3e_3 = e_4$		
$\mathbf{N}_{256}^{\alpha}$:	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_2e_2 = e_5$	
	$e_2e_4 = \alpha e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{257}$:	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_2e_3 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{258}$:	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{259}$:	$e_1e_2 = e_3$	$e_1e_4 = e_5$	$e_2e_4 = e_5$	
	$e_3e_3 = e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{260}^{\alpha}$:	$e_1e_2 = e_3$	$e_2e_2 = e_5$	$e_2e_3 = e_5$	
	$e_3e_3 = e_4$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{261}$:	$e_1e_2 = e_3$	$e_2e_2 = e_5$	$e_2e_4 = e_5$	
	$e_3e_3 = e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{262}$:	$e_1e_2 = e_3$	$e_2e_2 = e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{263}$:	$e_1e_2 = e_3$	$e_2e_2 = e_5$	$e_3e_3 = e_4$	
	$e_3e_4 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{264}$:	$e_1e_2 = e_3$	$e_2e_2 = e_5$	$e_3e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{265}$:	$e_1e_2 = e_3$	$e_2e_3 = e_5$	$e_3e_3 = e_4$	
	$e_3e_4 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{266}$:	$e_1e_2 = e_3$	$e_2e_3 = e_5$	$e_3e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{267}$:	$e_1e_2 = e_3$	$e_2e_4 = e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{268}$:	$e_1e_2 = e_3$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{269}$:	$e_1e_2 = e_3$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{270}$:	$e_1e_2 = e_3$	$e_3e_3 = e_4$	$e_4e_4 = e_5$	
$\mathbf{N}_{271}^{\alpha,\beta,\gamma,\mu}$:	$e_1e_1 = e_4 + e_5$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	$e_2e_2 = \beta e_5$
	$e_2e_3 = \gamma e_5$	$e_3e_3 = e_4$	$e_3e_4 = \mu e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{272}^{\alpha,\beta}$:	$e_1e_1 = e_4 + \alpha e_5$	$e_1e_2 = e_3$	$e_1e_3 = \beta e_5$	
	$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$	

$\mathbf{N}_{273}^{\alpha,\beta}$	:	$e_1e_1=e_4+\alpha e_5$	$e_1e_2=e_3$	$e_1e_3=\beta e_5$	
		$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{274}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_4+e_5$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=\beta e_5$
		$e_2e_4=\gamma e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{275}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$
		$e_2e_2=\alpha e_5$	$e_2e_3=\beta e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{276}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$
		$e_2e_3=\beta e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{277}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_2e_2=\alpha e_5$
		$e_2e_3=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$
$\mathbf{N}_{278}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{279}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{280}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=e_5$
		$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{281}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_3=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{282}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	
		$e_2e_4=e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{283}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	
		$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{284}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$
		$e_3e_3=e_4$	$e_3e_4=\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{285}^{\alpha}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_2e_3=e_5$	
		$e_3e_3=e_4$	$e_3e_4=\alpha e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{286}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_3e_3=e_4$	
		$e_3e_4=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{287}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_3e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{288}^{\alpha,\beta,\gamma,\mu,\nu}$	:	$e_1e_1=e_4+\alpha e_5$	$e_1e_2=e_3$	$e_1e_3=\beta e_5$	
		$e_2e_2=e_4+\gamma e_5$	$e_2e_3=\mu e_5$	$e_3e_3=e_4$	
		$e_3e_4=\nu e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{289}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_4+\alpha e_5$	$e_1e_2=e_3$	$e_1e_4=\beta e_5$	
		$e_2e_2=e_4+\gamma e_5$	$e_2e_4=\mu e_5$		
		$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{290}^{\alpha,\beta}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$
		$e_2e_2=e_4+\beta e_5$	$e_2e_3=\gamma e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{291}^{\alpha,\beta\neq0,\gamma}$	:	$e_1e_1=e_4$	$e_1e_2=e_3$	$e_1e_3=\alpha e_5$	$e_1e_4=\beta e_5$
		$e_2e_3=\gamma e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{292}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	
		$e_1e_4=\alpha e_5$	$e_2e_2=\beta e_5$	$e_2e_3=e_4$	
		$e_2e_4=e_5$	$e_3e_3=-\alpha e_5$		
$\mathbf{N}_{293}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$
		$e_2e_2=\alpha e_5$	$e_2e_3=e_4$	$e_3e_3=-e_5$	
$\mathbf{N}_{294}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_2e_2=\alpha e_5$
		$e_2e_3=e_4$	$e_2e_4=\beta e_5$	$e_3e_3=\gamma e_5$	$e_4e_4=e_5$
$\mathbf{N}_{295}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=\alpha e_5$	$e_2e_2=e_5$
		$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\beta e_5$	
$\mathbf{N}_{296}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_2=e_5$	$e_2e_3=e_4$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{297}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{298}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_3=e_4$	$e_3e_3=\alpha e_5$		

$\mathbf{N}_{299}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=e_4$
		$e_2e_4=\alpha e_4$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{300}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=e_4$
		$e_2e_4=\alpha e_4$	$e_3e_3=\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{301}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_2e_4=e_5$
$\mathbf{N}_{302}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	
		$e_2e_4=e_5$	$e_3e_3=e_5$		
$\mathbf{N}_{303}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{304}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{305}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	
		$e_3e_3=e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{306}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	
		$e_3e_3=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{307}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{308}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{309}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	
		$e_1e_4=e_5$	$e_2e_2=\alpha e_5$		
		$e_2e_3=e_4+\beta e_5$	$e_3e_3=-e_5$		
$\mathbf{N}_{310}^{\alpha,\beta\neq-1}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=e_5$
		$e_2e_2=\alpha e_5$	$e_2e_3=e_4$	$e_3e_3=\beta e_5$	
$\mathbf{N}_{311}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_1e_4=\alpha e_5$
		$e_2e_3=e_4+\beta e_5$	$e_2e_4=e_5$	$e_3e_3=\gamma e_5$	
$\mathbf{N}_{312}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=\alpha e_5$
		$e_2e_3=e_4+\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=\mu e_5$	$e_4e_4=e_5$
$\mathbf{N}_{313}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_4$	$e_2e_2=\alpha e_5$
		$e_2e_3=e_4$	$e_2e_4=\beta e_5$	$e_3e_3=\gamma e_5$	$e_3e_4=e_5$
$\mathbf{N}_{314}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=e_5$
		$e_2e_2=\alpha e_5$	$e_2e_3=\beta e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{315}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	$e_1e_4=\alpha e_5$
		$e_2e_2=\beta e_5$	$e_2e_3=\gamma e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{316}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_3=e_5$	
		$e_2e_2=\alpha e_5$	$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	
		$e_3e_3=e_4$	$e_3e_4=\mu e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{317}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{318}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_2=e_5$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{319}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=\alpha e_5$
		$e_2e_3=\beta e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{320}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	$e_2e_2=\alpha e_5$
		$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{321}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_1e_4=e_5$	
		$e_2e_3=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{322}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	
		$e_2e_3=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{323}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$
		$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{324}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_2=e_5$	$e_2e_3=\alpha e_5$
		$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$
$\mathbf{N}_{325}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$	
		$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{326}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_2=e_3$	$e_2e_3=e_5$	

		$e_2e_4 = \alpha e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{327}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_3 = e_5$	$e_2e_4 = \alpha e_5$
		$e_3e_3 = e_4$	$e_3e_4 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{328}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_4 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{329}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_4 = e_5$	
		$e_3e_3 = e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{330}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_2e_4 = e_5$	
		$e_3e_3 = e_4$	$e_3e_4 = \alpha e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{331}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_3e_3 = e_4$	$e_3e_4 = e_5$
$\mathbf{N}_{332}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_3e_3 = e_4$	
		$e_3e_4 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{333}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_3e_3 = e_4$	$e_4e_4 = e_5$
$\mathbf{N}_{334}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	$e_1e_4 = e_5$
		$e_2e_2 = e_4 + \beta e_5$	$e_2e_3 = \gamma e_5$	$e_3e_3 = e_4$	
$\mathbf{N}_{335}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	$e_1e_4 = \beta e_5$
		$e_2e_2 = e_4 + \gamma e_5$	$e_2e_3 = \mu e_5$	$e_2e_4 = e_5$	$e_3e_3 = e_4$
$\mathbf{N}_{336}^{\alpha,\beta,\gamma,\mu,\nu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_3 = \alpha e_5$	
		$e_2e_2 = e_4 + \beta e_5$	$e_2e_3 = \gamma e_5$	$e_2e_4 = \mu e_5$	
		$e_3e_3 = e_4$	$e_3e_4 = \nu e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{337}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_3$	$e_1e_4 = \alpha e_5$	
		$e_2e_2 = e_4 + \beta e_5$	$e_2e_3 = \gamma e_5$	$e_2e_4 = \mu e_5$	
		$e_3e_3 = e_4$	$e_3e_4 = e_5$		
$\mathbf{N}_{338}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = e_4$	$e_1e_4 = e_5$
		$e_2e_2 = e_3$	$e_2e_3 = -2e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{339}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = e_4$	$e_2e_2 = e_3$
		$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{340}^{\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = e_5$	$e_1e_3 = e_4$	$e_2e_2 = e_3$
		$e_2e_3 = \alpha e_5$	$e_3e_3 = \beta e_5$	$e_3e_4 = \gamma e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{341}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$		
$\mathbf{N}_{342}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	$e_2e_2 = e_3$
		$e_2e_3 = \alpha e_5$	$e_2e_4 = e_5$	$e_3e_3 = \beta e_5$	
$\mathbf{N}_{343}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_1e_4 = e_5$	
		$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$	$e_3e_3 = e_5$	
$\mathbf{N}_{344}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_2e_3 = e_5$	$e_2e_4 = e_5$	$e_3e_3 = \alpha e_5$	
$\mathbf{N}_{345}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_2e_3 = \alpha e_5$
		$e_2e_4 = e_5$	$e_3e_3 = \beta e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{346}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_2e_3 = e_5$	$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$	
$\mathbf{N}_{347}^{\alpha,\beta}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_2e_3 = e_5$
		$e_3e_3 = \alpha e_5$	$e_3e_4 = \beta e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{348}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_2e_4 = e_5$
$\mathbf{N}_{349}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_2e_4 = e_5$	$e_3e_3 = e_5$		
$\mathbf{N}_{350}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_3e_3 = e_5$	$e_3e_4 = e_5$		
$\mathbf{N}_{351}^{\alpha}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_3e_3 = \alpha e_5$	$e_3e_4 = e_5$	$e_4e_4 = e_5$	
$\mathbf{N}_{352}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	
		$e_3e_3 = e_5$	$e_4e_4 = e_5$		
$\mathbf{N}_{353}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_3e_4 = e_5$
$\mathbf{N}_{354}$	:	$e_1e_1 = e_2$	$e_1e_3 = e_4$	$e_2e_2 = e_3$	$e_4e_4 = e_5$

$\mathbf{N}_{355}^{\alpha\neq0,\beta}$	:	$e_1e_1=e_2$	$e_1e_3=e_4+\alpha e_5$		
		$e_1e_4=e_5$	$e_2e_2=e_3$		
		$e_2e_3=\beta e_5$	$e_3e_3=(\beta+2)e_5$		
$\mathbf{N}_{356}^{\alpha\neq0,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_4+\alpha e_5$		
		$e_2e_2=e_3$	$e_2e_3=\beta e_5$	$e_2e_4=e_5$	
		$e_3e_3=\gamma e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{357}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_3=e_4+\alpha e_5$		
		$e_2e_2=e_3$	$e_2e_3=\beta e_5$	$e_3e_3=\gamma e_5$	
		$e_3e_4=\mu e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{358}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=e_5$	
		$e_2e_2=e_3$	$e_2e_3=\alpha e_5$	$e_3e_3=\beta e_5$	
$\mathbf{N}_{359}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$
		$e_2e_3=\beta e_5$	$e_2e_4=e_5$	$e_3e_3=\gamma e_5$	
$\mathbf{N}_{360}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_2e_2=e_3$	$e_2e_3=\alpha e_5$
		$e_2e_4=\beta e_5$	$e_3e_3=\gamma e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{361}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$
		$e_2e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=\beta e_5$	
$\mathbf{N}_{362}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=\alpha e_5$	
		$e_2e_2=e_3$	$e_2e_3=e_4$	$e_2e_4=\beta e_5$	
		$e_3e_3=\gamma e_5$	$e_3e_4=\mu e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{363}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$
		$e_2e_3=e_4$	$e_2e_4=\beta e_5$	$e_3e_3=\gamma e_5$	$e_3e_4=e_5$
$\mathbf{N}_{364}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$
		$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\beta e_5$	
$\mathbf{N}_{365}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=e_5$	
		$e_2e_2=e_3$	$e_2e_3=e_4$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{366}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_2e_3=e_4$
		$e_1e_4=\alpha e_5$	$e_3e_3=\beta e_5$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$
$\mathbf{N}_{367}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=e_4$
$\mathbf{N}_{368}^{\alpha}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	
		$e_2e_3=e_4$	$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	
$\mathbf{N}_{369}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=e_4$
		$e_2e_4=\alpha e_5$	$e_3e_3=\beta e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{370}$	:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	
		$e_2e_3=e_4$	$e_3e_3=e_5$		
$\mathbf{N}_{371}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$		
$\mathbf{N}_{372}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	
		$e_2e_4=e_5$	$e_3e_3=\alpha e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{373}^{\alpha,\beta}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	$e_2e_4=e_5$
		$e_3e_3=\alpha e_5$	$e_3e_4=\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{374}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	
		$e_3e_3=e_5$	$e_3e_4=e_5$		
$\mathbf{N}_{375}^{\alpha}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	
		$e_3e_3=e_5$	$e_3e_4=\alpha e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{376}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{377}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	
		$e_3e_4=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{378}$	:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{379}^{\alpha,\beta,\gamma}$	:	$e_1e_1=e_2$	$e_1e_2=\alpha e_5$	$e_1e_3=e_4$	
		$e_1e_4=e_5$	$e_2e_2=e_3$		
		$e_2e_3=e_4+\beta e_5$	$e_3e_3=\gamma e_5$		
$\mathbf{N}_{380}^{\alpha,\beta,\gamma,\mu}$	:	$e_1e_1=e_2$	$e_1e_2=\alpha e_5$	$e_1e_3=e_4$	

	$e_1e_4=\beta e_5$	$e_2e_2=e_3$	$e_2e_3=e_4$	
	$e_2e_4=\gamma e_5$	$e_3e_3=\mu e_5$	$e_3e_4=e_5$	
$\mathbf{N}_{381}^{\alpha,\beta,\gamma,\mu,\nu}$:	$e_1e_1=e_2$	$e_1e_2=\alpha e_5$	$e_1e_3=e_4$	$e_2e_2=e_3$
	$e_2e_3=e_4+\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=\mu e_5$	
	$e_3e_4=\nu e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{382}^{\alpha,\beta,\gamma}$:	$e_1e_1=e_2$	$e_1e_3=e_4$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$
	$e_2e_3=e_4+\beta e_5$	$e_2e_4=e_5$	$e_3e_3=\gamma e_5$	
$\mathbf{N}_{383}^{\alpha,\beta}$:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$
	$e_2e_2=e_3$	$e_2e_3=\beta e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{384}^{\alpha,\beta,\gamma,\mu}$:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_3=\alpha e_5$	
	$e_2e_2=e_3$	$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	
	$e_3e_3=e_4$	$e_3e_4=\mu e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{385}^{\alpha,\beta,\gamma}$:	$e_1e_1=e_2$	$e_1e_2=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$
	$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{386}^{\alpha}$:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=e_5$	
	$e_2e_2=e_3$	$e_2e_3=\alpha e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{387}^{\alpha,\beta}$:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$
	$e_2e_3=\beta e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{388}^{\alpha,\beta,\gamma}$:	$e_1e_1=e_2$	$e_1e_3=e_5$	$e_2e_2=e_3$	$e_2e_3=\alpha e_5$
	$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=\gamma e_5$	$e_4e_4=e_5$
$\mathbf{N}_{389}$:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_3e_3=e_4$
$\mathbf{N}_{390}$:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	
	$e_2e_3=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{391}^{\alpha}$:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	
	$e_2e_3=\alpha e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$	
$\mathbf{N}_{392}^{\alpha,\beta}$:	$e_1e_1=e_2$	$e_1e_4=e_5$	$e_2e_2=e_3$	$e_2e_3=\alpha e_5$
	$e_2e_4=\beta e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{393}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_5$	
	$e_2e_4=e_5$	$e_3e_3=e_4$		
$\mathbf{N}_{394}^{\alpha}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_5$	
	$e_2e_4=\alpha e_5$	$e_3e_3=e_4$	$e_3e_4=e_5$	
$\mathbf{N}_{395}^{\alpha,\beta}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_3=e_5$	$e_2e_4=\alpha e_5$
	$e_3e_3=e_4$	$e_3e_4=\beta e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{396}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{397}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_4=e_5$	
	$e_3e_3=e_4$	$e_3e_4=e_5$		
$\mathbf{N}_{398}^{\alpha}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_2e_4=e_5$	
	$e_3e_3=e_4$	$e_3e_4=\alpha e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{399}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_3e_3=e_4$	$e_3e_4=e_5$
$\mathbf{N}_{400}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_3e_3=e_4$	
	$e_3e_4=e_5$	$e_4e_4=e_5$		
$\mathbf{N}_{401}$:	$e_1e_1=e_2$	$e_2e_2=e_3$	$e_3e_3=e_4$	$e_4e_4=e_5$
$\mathbf{N}_{402}^{\alpha,\beta,\gamma}$:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=\alpha e_5$	$e_1e_4=\beta e_5$
	$e_2e_2=e_3$	$e_2e_3=\gamma e_5$	$e_2e_4=e_5$	$e_3e_3=e_4$
$\mathbf{N}_{403}^{\alpha,\beta,\gamma}$:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=\alpha e_5$	$e_1e_4=e_5$
	$e_2e_2=e_3$	$e_2e_3=\beta e_5$	$e_3e_3=e_4+\gamma e_5$	
$\mathbf{N}_{404}^{\alpha,\beta,\gamma,\mu,\nu}$:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_3=\alpha e_5$	
	$e_2e_2=e_3$	$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	
	$e_3e_3=e_4+\mu e_5$	$e_3e_4=\nu e_5$	$e_4e_4=e_5$	
$\mathbf{N}_{405}^{\alpha,\beta,\gamma,\mu}$:	$e_1e_1=e_2$	$e_1e_2=e_4$	$e_1e_4=\alpha e_5$	$e_2e_2=e_3$
	$e_2e_3=\beta e_5$	$e_2e_4=\gamma e_5$	$e_3e_3=e_4+\mu e_5$	$e_3e_4=e_5$
$\mathbf{N}_{406}^{\lambda,\alpha,\beta,\gamma}$:	$e_1e_1=e_2$	$e_1e_2=\lambda e_4+\alpha e_5$		

		$e_1e_3 = \beta e_5$	$e_1e_4 = e_5$	$e_2e_2 = e_3$	
		$e_2e_3 = e_4 + \gamma e_5$	$e_3e_3 = e_4$		
$\mathbf{N}_{407}^{\lambda,\alpha,\beta,\gamma,\mu,\nu}$	:	$e_1e_1 = e_2$	$e_1e_2 = \lambda e_4 + \alpha e_5$		
		$e_1e_3 = \beta e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4 + \gamma e_5$	
		$e_2e_4 = \mu e_5$	$e_3e_3 = e_4$	$e_3e_4 = \nu e_5$	$e_4e_4 = e_5$
$\mathbf{N}_{408}^{\lambda,\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = \lambda e_4 + \alpha e_5$		
		$e_1e_4 = \beta e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4 + \gamma e_5$	
		$e_2e_4 = \mu e_5$	$e_3e_3 = e_4$	$e_3e_4 = e_5$	
$\mathbf{N}_{409}^{\lambda,\alpha,\beta,\gamma}$	:	$e_1e_1 = e_2$	$e_1e_2 = \lambda e_4$	$e_1e_3 = \alpha e_5$	
		$e_1e_4 = \beta e_5$	$e_2e_2 = e_3$	$e_2e_3 = e_4 + \gamma e_5$	
		$e_2e_4 = e_5$	$e_3e_3 = e_4$		

REFERENCES

[1] Yo. Cabrera Casado, M. Siles Molina and M. V. Velasco, Classification of three-dimensional evolution algebras, *Linear Algebra Appl.*, **524** (2017), 68–108.

[2] E. M. Cañete and A. Kh. Khudoyberdiyev, The classification of 4-dimensional Leibniz algebras, *Linear Algebra Appl.*, **439** (2013), 273–288.

[3] S. Cicalò, W. A. de Graaf and C. Schneider, Six-dimensional nilpotent Lie algebras, *Linear Algebra Appl.*, **436** (2012),163–189.

[4] I. Darijani and H. Usefi, The classification of 5-dimensional p-nilpotent restricted Lie algebras over perfect fields, I., *J. Algebra*, **464** (2016), 97–140.

[5] E. Darpö and A. Rochdi, Classification of the four-dimensional power-commutative real division algebras, *Proc. Roy. Soc. Edinburgh Sect. A*, **141** (2011), 1207–1223.

[6] W. A. de Graaf, Classification of 6-dimensional nilpotent Lie algebras over fields of characteristic not 2, *J. Algebra*, **309** (2007), 640–653.

[7] E. Dieterich and J. Öhman, On the classification of 4-dimensional quadratic division algebras over square-ordered fields, *J. London Math. Soc. (2)*, **65** (2002), 285–302.

[8] A. Fernández Ouaridi, I. Kaygorodov, M. Khrypchenko and Yu. Volkov, Degenerations of nilpotent algebras, *J. Pure Appl. Algebra*, **226** (2022), Paper No. 106850, 21 pp.

[9] A. S. Hegazi and H. Abdelwahab, Classification of five-dimensional nilpotent Jordan algebras, *Linear Algebra Appl.*, **494** (2016), 165–218.

[10] A. S. Hegazi, H. Abdelwahab and A. J. Calderón Martín, The classification of n-dimensional non-Lie Malcev algebras with $(n-4)$-dimensional annihilator, *Linear Algebra Appl.*, **505** (2016), 32–56.

[11] D. Jumaniyozov, I. Kaygorodov and A. Khudoyberdiyev, The algebraic classification of nilpotent commutative $\mathfrak{CD}$-algebras, *Communications in Algebra*, **49** (2021), 1464–1494.

[12] I. Kaygorodov, M. Khrypchenko and S. A. Lopes, The algebraic and geometric classification of nilpotent anticommutative algebras, *Journal of Pure and Applied Algebra*, **224** (2020), 106337, 32 pp.

[13] I. Kaygorodov, M. Khrypchenko and S. Lopes, The algebraic classification of nilpotent algebras, arXiv:2012.00525.

[14] I. Kaygorodov, M. Khrypchenko and Yu. Popov, The algebraic and geometric classification of nilpotent terminal algebras, *J. Pure Appl. Algebra*, **225** (2021), 106625, 41 pp.

[15] I. Kaygorodov, I. Rakhimov and Sh. K. Said Husain, The algebraic classification of nilpotent associative commutative algebras, *J. Algebra Appl.*, **19** (2020), 2050220, 14 pp.

[16] I. Kaygorodov and Yu. Volkov, The variety of 2-dimensional algebras over an algebraically closed field, *Canad. J. Math.*, **71** (2019), 819–842.

[17] Yu. Kobayashi, K. Shirayanagi, S.-Ei. Takahasi and M. Tsukada, Classification of three-dimensional zeropotent algebras over an algebraically closed field, *Comm. Algebra*, **45** (2017), 5037–5052.

[18] G. Mazzola, Generic finite schemes and Hochschild cocycles, *Comment. Math. Helv.*, **55** (1980), 267–293.

[19] H. P. Petersson, The classification of two-dimensional nonassociative algebras, *Results Math.*, **37** (2000), 120–154.

[20] T. Skjelbred and T. Sund, Sur la classification des algebres de Lie nilpotentes, *C. R. Acad. Sci. Paris Sér. A-B*, **286** (1978), A241–A242.

Received February 2021; revised July 2021; early access September 2021.

E-mail address: jumaniyozovdoston50@gmail.com
E-mail address: kaygorodov.ivan@gmail.com
E-mail address: abror.khudoyberdiyev@mathinst.uz