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# ДОКЛАДЫ Академии наук Республики Узбекистан



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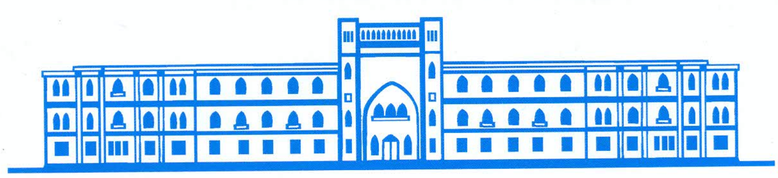

$$\left\|\overset{\circ}{\ell}\right\|^2 = (-1)^{m+1}\left[\frac{B_{2m}h^{2m}}{(2m)!} + \frac{2h^{2m+1}}{(2m)!}\sum_{k=1}^{m-1}\sum_{i=0}^{2m} d_k \frac{q_k + q_k^{N+i}(-1)^{i+1}}{(q_k-1)^{i+1}}\Delta^i 0^{2m}\right],$$

*где* $d_k$ *определяются из системы* (12)-(14), $B_{2m}$ *– числа Бернулли,* $q_k$ *– корни многочлена Эйлера степени* $2m-2$ [6], $|q_k|<1$.

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Ф.А. Нуралиев

ҲОСИЛАЛИ СОНЛИ ИНТЕГРАЛЛАШ ФОРМУЛАЛАРИ

Ушбу ишда $L_2^{(m)}(0,1)$ фазосида оптимал квадратур формулалар қуриш масаласи қаралган. Бунда квадратур йиғинди интеграл остидаги функциянинг тугун нуқталардаги қийматларидан ва унинг биринчи, учинчи ва бешинчи тартибли ҳосилаларининг интеграллаш интервали охирларидаги қийматларидан ташкил топган. Ишда оптимал коэффициентлар топилиб, хатолик функционали нормаси ҳисобланган.



F.A. Nuraliev

FORMULA OF NUMERICAL INTEGRATION WITH DERIVATIVES

In this paper in the space $L_2^{(m)}(0,1)$ the problem of construction of optimal quadrature formulas is considered. Here the quadrature sum consists on values of integrand at nodes and values of first, third, fifth derivatives of integrand at the end points of integration interval. In the work the optimal coefficients are found and norm of the error functional is calculated



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# CRITICAL VALUES OF A NEAREST-NEIGHBOR SYSTEM ON A CAYLEY TREE

*(Представлено акад. АН РУз Ш.А. Аюповым)*

For several models on Cayley trees (such as Ising and Potts models with competing interactions) the full description of translation-invariant and periodic Gibbs measures has been obtained (see [2]).

The Cayley tree (Bethe lattice) $\Im^k$ of order $k \geq 2$ is an infinite tree, i.e., a graph without cycles, such that exactly $k+1$ edges originate from each vertex (see [1]). Let $\Im^k = (V, L)$ where $V$ is the set of vertices and $L$ the set of edges. Two vertices $x$ and $y$ are called *nearest neighbors* if there exists an edge $l \in L$ connecting them and we denote $l = <x, y>$.

It is well-known that there exists a one-to-one correspondence between the set $V$ of vertices of the Cayley tree of order $k \geq 2$ and the group $G_k$ the free product of $k+1$ cyclic groups of second order with generators $a_1, a_2, ..., a_{k+1}$.

A configuration $\sigma$ on $V$ is then defined as a function $x \in V \to [0,1]$; the set of all configurations is $[0,1]^V$. For $A \subset V$ a configuration $\sigma_A$ on $A$ is an arbitrary function $\sigma_A : A \to [0,1]$. Denote $\Omega_A = [0,1]^A$ the set of all configurations on $A$.

We consider a model where the spin takes values in the set [0,1], and is assigned to the vertices of the Cayley tree.

The (formal) Hamiltonian of the model is:

$$H(\sigma) = -J \sum_{<x,y>\in L} \xi_{\sigma(x)\sigma(y)} \tag{1}$$

where $J \in R \setminus \{0\}$ and $\xi : (u,v) \in [0,1]^2 \to \xi_{uv} \in R$ is a given bounded, measurable function [1].

Let $\lambda$ be the Lebesgue measure on [0,1]. On the set of all configurations on $A$ the a priori measure $\lambda_A$ is introduced as the $A$ fold product of the measure $\lambda$. Here and further on $|A|$ denotes the cardinality of $A$. We consider a standard sigma-algebra $B$ of subsets of $\Omega = [0,1]^V$ generated by the measurable cylinder subsets. A probability measure $\mu$ on $\Omega$ is called a Gibbs measure (with Hamiltonian $H$) if it satisfies the Dobrushin-Lanford-Ruelle (DLR) equation, namely for any $n = 1,2,...$ and $\sigma_n \in \Omega_{V_n}$:

$$\mu(\{\sigma \in \Omega : \sigma|_{V_n} = \sigma_n\}) = \int_\Omega \mu(d\omega) \nu^{V_n}_{\omega|W_{n+1}},$$

where $\nu^{V_n}_{\omega|W_{n+1}}$ is the conditional Gibbs density

$$\nu^{V_n}_{\omega|W_{n+1}}(\sigma_n) = \frac{1}{Z_n(\omega|W_{n+1})} \exp(-\beta H(\sigma_n \,||\, \omega|_{W_{n+1}})),$$

and $\beta = \frac{1}{T}$, $T > 0$ is temperature.

Let $L_n = \{<x,y> \in L : x, y \in V_n\}$ and $\Omega_{V_n}$ is the set of configurations in $V_n$ (and $\Omega_{V_n}$ that in $W_n$). Furthermore, $\sigma|_{V_n}$ and $\omega|_{W_n}$ denote the restrictions of configurations $\sigma, \omega \in \Omega$ to $V_n$ and $W_{n+1}$, respectively. Next, $\sigma_n : x \in V_n \to \sigma_n(x)$ is a configuration in $V_n$ and $H(\sigma_n \,||\, \omega|_{W_{n+1}})$ is defined as the sum $H(\sigma_n) + U(\sigma_n, \omega|_{W_{n+1}})$ where

$$H(\sigma_n) = -J \sum_{<x,y>\in L_n} \xi_{\sigma_n(x)\sigma_n(y)},$$

$$U(\sigma_n, \omega|_{W_{n+1}}) = -J \sum_{\substack{<x,y>: \\ x\in V_n, y \in W_{n+1}}} \xi_{\sigma_n(x)\omega(y)}.$$

Finally, $Z_n(\omega|W_{n+1})$ stands for the partition function in $V_n$, with the boundary condition $\omega|_{W_{n+1}}$:

$$Z_n(\omega|_{W_{n+1}}) = \int_{\Omega_{V_n}} \exp(-\beta H(\tilde{\sigma}_n \,||\, \omega|_{W_{n+1}})) \lambda_{V_n}(d\tilde{\sigma}_n).$$

Write $x<y$ if the path from $x^0$ to $y$ goes through $x$. Call vertex $y$ a direct successor of $x$ if $x>y$ and $x,y$ are nearest neighbors. Denote by $S(x)$ the set of direct successors of $x$. Observe that any vertex $x \neq x^0$ has $k$ direct successors and $x^0$ has $k+1$.

Let $h : x \in V \to h_x = (h_{t,x}, t \in [0,1]) \in R^{[0,1]}$ be a mapping of $x \in V \setminus \{x^0\}$. Given $n=1,2,...$ consider the probability distribution $\mu^{(n)}$ on $\Omega_{V_n}$ defined by

$$\mu^{(n)}(\sigma_n) = Z_n^{-1} \exp(-\beta H(\sigma_n) + \sum_{x\in W_n} h_{\sigma(x),x}), \tag{2}$$

Here, as before, $\sigma_n : x \in V_n \to \sigma(x)$ and $Z_n$ is the corresponding partition function:

$$Z_n = \int_{\Omega_{V_n}} \exp(-\beta H(\tilde{\sigma}_n) + \sum_{x\in W_n} h_{\tilde{\sigma}(x),x}) \lambda_{V_n}(d\tilde{\sigma}_n). \tag{3}$$

The probability distributions $\mu^{(n)}$ are called compatible if for any $n \geq 1$ and $\sigma_{n-1} \in \Omega_{V_{n-1}}$:

$$\int_{\Omega_{W_n}} \mu^{(n)}(\sigma_{n-1} \vee \omega_n) \lambda_{W_n}(d(\omega_n)) = \mu^{(n-1)}(\sigma_{n-1}). \tag{4}$$

Here $\sigma_{n-1} \vee \omega_n \in \Omega_{V_n}$ is the concatenation of $\sigma_{n-1}$ and $\omega_n$. In this case.

**Proposition 1. [3].** *The probability distributions $\mu^{(n)}(\sigma_n)$, $n = 1,2,...$, in (2) are compatible iff for any $x \in V \setminus \{x^0\}$ the following equation holds:*

$$f(t,x) = \prod_{y\in S(x)} \frac{\int_0^1 \exp(J\beta\xi_{tu}) f(u,y) du}{\int_0^1 \exp(J\beta\xi_{0u}) f(u,y) du}. \tag{5}$$

*Here, and below* $f(t,u)=\exp(h_{t,x}-h_{0,x}),\ t\in[0,1]$ *and* $du=\lambda(du)$ *is the Lebesgue measure.*

From Proposition 1 it follows that for any $h=\{h_x\in R^{[0,1]},\ x\in V\}$ satisfying (5) there exists a unique Gibbs measure $\mu$ and vice versa. However, the analysis of solutions to (5) is not easy. This difficulty depends on the given function $\xi$.

Let $\xi_{tu}$ be a continuous function; we are going to construct functions $\xi_{tu}$ under which the equation (5) has at least two solutions in the class of translation-invariant functions $f(t,x)$, i.e., $f(t,x)=f(t)$, for any $x\in V$. For such functions the equation (5) can be written as

$$f(t)=\left(\frac{\int_0^1 K(t,u)f(u)du}{\int_0^1 K(0,u)f(u)du}\right)^k, \tag{6}$$

where $K(t,u)=\exp(J\beta\xi_{tu}),\ \ f(t)>0,\ t,u\in[0,1]$.

We put

$$C^+[0,1]=\{f\in C[0,1];f(x)\geq 0\}.$$

We are interested in positive continuous solutions of (6).

For every $k\in\mathrm{N}$ we consider an integral operator $H_k$ acting in the cone $C^+[0,1]$ as

$$(H_kf)(t)=\int_0^1 K(t,u)f^k(u)du,\quad k\in\mathrm{N}.$$

The operator $H_k$ is called Hammerstein's integral operator of order $k$. This operator is well known to generate ill-posed problems. Clearly, if $k\geq 2$ then $H_k$ is a nonlinear operator.

It is known that the set of translation-invariant Gibbs measures of the model (1) is described by the fixed points of Hammerstein's operator (see [6]).

Let $k\geq 2$ in the model (1) and

$$\xi_{t,u}=\xi_{t,u}(\theta,\beta)=\frac{1}{\beta}\ln\left(1+\theta\sqrt[3]{4\left(t-\frac{1}{2}\right)\left(u-\frac{1}{2}\right)}\right),\quad t,u\in[0,1]$$

where $0\leq\theta<1$. Then for the Kernel $K(t,u)$ of Hammerstein's operator $H_k$ we have

$$K(t,u)=1+\theta\sqrt[3]{4\left(t-\frac{1}{2}\right)\left(u-\frac{1}{2}\right)}.$$

We define the operator $V_k:(x,y)\in R^2\to(x',y')\in R^2$ for even $k\geq 2$:

$$V_k(x,y)=\begin{cases} x'=3\sum\limits_{l=0,2,\ldots,k}\binom{k}{l}x^l(\sqrt[3]{2}\theta y)^{k-l}A_k(l)\\ y'=3\sum\limits_{l=1,3,\ldots,k-1}\binom{k}{l}x^l(\sqrt[3]{2}\theta y)^{k-l}B_k(l)\end{cases} \tag{7}$$

and for odd $k\geq 3$:

$$V_k(x,y)=\begin{cases} x'=3\sum\limits_{l=0,2,\ldots,k}\binom{k}{l}x^l(\sqrt[3]{2}\theta y)^{k-l}A_k(l)\\ y'=3\sum\limits_{l=1,3,\ldots,k-1}\binom{k}{l}x^l(\sqrt[3]{2}\theta y)^{k-l}B_k(l)\end{cases} \tag{8}$$

Here we denote

$$A_k(l):=\frac{1}{k+1-l}-\frac{2}{(k+2-l)(k+1-l)}+\frac{3}{(k+3-l)(k+2-l)(k+1-l)},$$

$$B_k(l):=\frac{\sqrt[3]{4}}{2(k+1-l)}-\frac{3\sqrt[3]{4}}{2(k+2-l)(k+1-l)}+\frac{3\sqrt[3]{4}}{(k+3-l)(k+2-l)(k+1-l)}-$$

$$-\frac{3\sqrt[3]{4}}{(k+4-l)(k+3-l)(k+2-l)(k+1-l)}.$$

**Theorem 1.** *A function $\varphi \in C[0,1]$ is a solution of Hammerstein's equation*

$$(H_k f)(t) = f(t)$$

iff $\varphi(t)$ has the following form

$$\varphi(t) = x' + y'\theta\sqrt[3]{4\left(t - \frac{1}{2}\right)},$$

*where $(x', y') \in R^2$ is a fixed point of the operator $V_k$ (7) and (8).*

**Theorem 2.** *a) If $0 \le \theta \le \frac{5}{3k}$, then Hammerstein's operator $H_k$ has a unique (nontrivial) positive fixed point in $C[0,1]$;*

*b) If $\frac{5}{3k} < \theta < 1$, then there are exactly three positive fixed points in $C[0,1]$ of Hammerstein's operator.*

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КЭЛИ ДАРАХТИДАГИ ЯҚИН-ҚЎШНИЛАР СИТЕМАСИНИНГ КРИТИК ҚИЙМАТЛАРИ

Ушбу мақолада иккинчи ва ундан юқори тартибли Кэли дарахтида аниқланган энг яқин ўзаро таъсирли ҳамда спин қийматлари [0,1] (саноқсиз) тўпламдан бўлган модел қаралган. Шунингдек, агар $0 \le \theta \le \frac{5}{3k}$ (бу ерда $\theta$ моделнинг параметри) бўлса, ягона трансляцион-инвариант Гиббс ўлчови мавжудлиги; агар $\frac{5}{3k} < \theta < 1$ бўлса, 3 та трансляцион-инвариант Гиббс ўлчовлари мавжудлиги кўрастилагн.

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КРИТИЧЕСКИЕ ЗНАЧЕНИЯ БЛИЖАЙШИХ СОСЕДЕЙ СИСТЕМЫ НА ДЕРЕВЕ КЭЛИ

В этой статье рассматривается модель взаимодействия ближайших соседей и (несчетного) набора [0,1] спиновых значений на дереве Кэли $\Im^k$ порядка $k \ge 2$. Доказано, что если $0 \le \theta \le \frac{5}{3k}$, (где параметр $\theta$ модели), то для модели существует единственная трансляцион-инвариантная мера Гиббса; если $\frac{5}{3k} < \theta < 1$, то существует три трансляцион-инвариантные меры Гиббса.

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## СРАВНЕНИЕ РИСКОВЫХ СИТУАЦИЙ В АКТУАРНОЙ (СТРАХОВОЙ) МАТЕМАТИКЕ

Рассмотрим некоторую страховую организацию (компанию), выпустившую и продавшую $n$ страховых полисов. Пусть резервный капитал (нелегальный капитал) компании равен $S$. Предположим, что каждый страховой контракт влечет за собой страховые выплаты клиентам, которые являются независимыми случайными величинами (с.в.). Обозначим с.в. выплаты $i$ – му клиенту через $X_i$. Тогда