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On Leibniz algebras with only inner derivations

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Barcha differensiallashlari ichki bo‘lgan Leybnits algebralari
Ushbu ishda mukammal Leybnits algebralarining ba’zi xossalarini qaraymiz. Mukammal Li algebralari uchun olingan ba’zi natijalarni Leybnits algebralari uchun kengaytiramiz.
Kalit so‘zlar: Leybnits algebrasi; ideal; nilradikal; radikal; markaz; differensiallash.

Алгебры Лейбница только с внутренним дифференцированием
В этой статье мы рассматриваем некоторые свойства совершенные алгебры Лейбница. Мы распространяем некоторые результаты, полученные для совершенных алгебр Ли, на случай алгебр Лейбница.
Ключевые слова: Алгебра Лейбница; идеал; нильрадикал; радикал; центр; дифференцирование.

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Introduction

A Lie algebra is called complete if its center is zero, and all its derivations are inner. The definition of complete Lie algebras was given by N. Jacobson in 1962 [12]. The first important result of complete Lie algebras first appeared in 1951 [11], in the context of Schenkman's theory of subinvariant Lie algebras. In recent years, different authors have concentrated on classifications and structural properties of complete Lie algebras.

It is well known that semisimple Lie algebras over field of characteristic 0, the Borel subalgebras, and the parabolic subalgebras of complex semisimple Lie algebras are complete Lie algebras. It has been proved by E. V. Schenkman that the holomorph of an abelian Lie algebra over the complex field C is a complete Lie algebra. A complete Lie algebra is called a simple complete Lie algebra if not any of its non-trivial ideals are complete. D. J. Meng [8] has proved that any finite dimensional complete

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Lie algebra can be decomposed into the direct sum of simple complete ideals, and the decomposition is unique up to the order of the ideals. He has studied some complete Lie algebras with commutative nilpotent radical and other complete Lie algebras whose nilpotent radicals are the direct sum of abelian Lie algebras and Heisenberg algebras in [8], [10]. Heisenberg algebras play an important role in physics. C. P. Jiang, D. J. Meng and S. Q. Zhang [6] have given the derivation algebra $DerH$ and the holomorph $h(H)$ of the finite dimensional Heisenberg algebra H over the complex field and have also given the derivation algebra $Der(h(H))$ of $h(H)$. They proved that $DerH$ was a simple complete Lie algebra, $h(H)$ was not a complete Lie algebras, but its derivation algebra $Derh(H)$ was a simple complete Lie algebra. Therefore, they have obtained two important classes of complete Lie algebras.

In [9] solvable complete Lie algebras were studied by D. J. Meng and L. S. Zhu. However, up to now there are a great deal of complete Lie algebras unknown. So, looking for complete Lie algebras is still an important task. In 2002 Y. C. Gao and D. J. Meng [5] first have given a necessary and sufficient condition for some solvable Lie algebras with l-step nilpotent radicals to be complete and a method to construct non-solvable complete Lie algebras.

A comprehensive study of the Lie algebra theory resulted in a number of beautiful results and generalizations. In particular, Loday introduced in [7] a non skew-symmetric analogue of a Lie algebra, called Leibniz algebra.

A simple, but yet productive property from Lie theory, namely the fact that the right multiplication operator on an element of the algebra is a derivation, can also be taken as a defining property for a Leibniz algebra. In the last years, Leibniz algebras have been under active research; among the numerous papers devoted to this subject, we can find some (co)homology and deformations properties, results on various types of decompositions, structure of solvable and nilpotent Leibniz algebras and classifications of some classes of graded nilpotent Leibniz algebras. Also, many results of theory of Lie algebras have been extended to the Leibniz algebras case. For instance, the classical results on Cartan subalgebras, Levi decomposition, Killing form, Engel's theorem, properties of solvable algebras with a given nilradical and others from the theory of Lie algebras are also true for Leibniz algebras. Recently, D. Barnes proved an analogue of Levi's theorem for the case of Leibniz algebras [3]; namely, a Leibniz algebra L is decomposed into a semidirect sum of its solvable radical and a semisimple Lie subalgebra, $L = S\dot{+}R$. He also presents an example in which two semisimple Lie subalgebras corresponding to different decompositions are not conjugate by an inner automorphism.

The aim of this article is to discuss complete Leibniz algebras. In this work we consider some properties of complete Leibniz algebras. We extend some results obtained for complete Lie algebras to the case of Leibniz algebras.

Throughout the paper we denote by $\mathbb{F}$ a field of characteristic zero and by L a finite dimensional Leibniz algebra over $\mathbb{F}$.

Preliminaries

Definition 1. A vector space with bilinear bracket $(L, [-,-])$ over a field $\mathbf{F}$ is called a Leibniz algebra if for any $x, y, z \in L$ the so-called Leibniz identity

$$\big[x,[y,z]\big] = \big[[x,y],z\big] - \big[[x,z],y\big]$$

holds.

Definition 2. For a given Leibniz algebra $(L, [-,-])$ the sequences of two-sided ideals defined recursively as follows:

$$L^1 = L,\ L^{k+1} = [L^k, L],\ k \geq 1,$$

$$L^{[1]} = L,\ L^{[s+1]} = [L^{[s]}, L^{[s]}],\ s \geq 1,$$

are said to be *the lower central* and *the derived series* of L, respectively.

DEFINITION 3. A Leibniz algebra L is said to be nilpotent (respectively, solvable), if there exists $n \in \mathbb{N}$ ($m \in \mathbb{N}$) such that $L^n = 0$ (respectively, $L^{[m]} = 0$). The minimal number n (respectively, m) with such property is said to be the index of nilpotency (respectively, of solvability) of the algebra L.

Evidently, the index of nilpotency of an n-dimensional nilpotent algebra is not greater than $n+1$.

DEFINITION 3. An n-dimensional Leibniz algebra L is said to be null-filiform if $\dim L^i = n+1-i,\ 1 \leq i \leq n+1$.

Evidently, null-filiform Leibniz algebras have maximal index of nilpotency.

Theorem 1.[1] *An arbitrary n-dimensional null-filiform Leibniz algebra is isomorphic to the algebra*

$$NF_n: \quad [e_i, e_1] = e_{i+1}, \quad 1 \leq i \leq n-1,$$

where $\{e_1, e_2, \dots, e_n\}$ is a basis of the algebra NF_n.

From this theorem it is easy to see that a nilpotent Leibniz algebra is null-filiform if and only if it is a one-generated algebra, i.e. an algebra generated by a simple element. Note that this notion has no sense in Lie algebras case, because they are at least two-generated.

DEFINITION 4. The maximal nilpotent (respectively, solvable) ideal of a Leibniz algebra is called the nilradical (respectively, radical) of the algebra.

Notice that the nilradical is not the radical in the sense of Kurosh, because the quotient Leibniz algebra by its nilradical may contain a nilpotent ideal (see [12]).

Theorem 2. [4] *Let R be a solvable Leibniz algebra whose nilradical is NF_n. Then there exists a basis $\{e_1, e_2, \dots, e_n, x\}$ of the algebra R such that the multiplication table of R with respect to this basis has the following form:*

$$\begin{cases} [e_i, e_1] = e_{i+1}, & 1 \leq i \leq n-1, \\ [x, e_1] = e_1, & \\ [e_i, x] = -ie_i, & 1 \leq i \leq n. \end{cases}$$

We recall an analogue of Levi's theorem for Leibniz algebras given in [3].

Theorem 3. *Let L be a finite-dimensional Leibniz algebra over a field of characteristic zero and let R be its solvable radical. Then there exists a semisimple Lie subalgebra S of L such that $L = S \dot{+} R$.*

The subalgebra S of the above theorem, similarly to Lie algebras theory, is called a *Levi subalgebra* of the Leibniz algebra L.

DEFINITION 5. A linear map $d\colon L \to L$ of a Leibniz algebra $(L, [-,-])$ is said to be a derivation if for all $x, y \in L$, the following condition holds:

$$d([x, y]) = [d(x), y] + [x, d(y)]$$

The set of all derivations of L (denoted by $Der(L)$) forms a Lie algebra with respect to the commutator.

For a given element x of a Leibniz algebra L, the right multiplication operator $R_x\colon L \to L$, defined by $R_x(y) = [y, x]$, is a derivation. In fact, a Leibniz algebra is characterized by this property of the right multiplication operators. As in Lie case this kind derivations are said to be *inner derivations*. Let the set of all inner derivations of a Leibniz algebra L denote by $R(L)$, i.e. $R(L) = \{R_x \mid x \in L\}$.

DEFINITION 6. The set $Z(L) = \{z \in L : [x, z] = [z, x] = 0,\ \forall x \in L\}$ is called *the center* of L.

DEFINITION 7. A Lie algebra G is called *complete* if its derivations are all inner and its center is 0.

Proposition 1.[12] *If R is complete and an ideal in G Lie algebra, then $G = R \oplus S$ where S is an ideal.*

In this paper we consider the notion of complete Leibniz algebras as given in [2].

DEFINITION 8.[2] A Leibniz algebra L is called *complete* if $Z(L) = 0$ and all derivations of L are inner.

The solvable Leibniz algebra R in Theorem 2 was shown to be complete in [2] by Ancochea Bermudez J. M. and Campoamor-Stursberg R.

Main results

We have the following result:

Theorem 4. Let L be a Leibniz algebra and $L = R\dot{+}sl_2$ be its Levi decomposition, where R is a complete solvable ideal whose nilradical is NF_n. Then $L = R \oplus sl_2$ in other words, L is the direct sum of ideals.

Proof. Let L be a Leibniz algebra, such that $L = R\dot{+}sl_2$, be its Levi decomposition, where R is a solvable ideal whose nilradical is NF_n. Then there exists a basis $\{e, h, f, e_1, e_2, \dots, e_n, x\}$ of the algebra L such that the table of multiplication in L has the following form:

$$\begin{array}{lll}
[e,h] = 2e, & [h,f] = 2f, & [e,f] = h, \\
[h,e] = -2e, & [f,h] = -2f, & [f,e] = -h, \\
[e,e_i] = \sum\limits_{j=1}^{n} \alpha_{ij} e_j + \alpha_i x, & [h,e_i] = \sum\limits_{j=1}^{n} \beta_{ij} e_j + \beta_i x, & [f,e_i] = \sum\limits_{j=1}^{n} \gamma_{ij} e_j + \gamma_i x, \\
[e_i,e] = \sum\limits_{j=1}^{n} \alpha'_{ij} e_j + \alpha'_i x, & [e_i,h] = \sum\limits_{j=1}^{n} \beta'_{ij} e_j + \beta'_i x, & [e_i,f] = \sum\limits_{j=1}^{n} \gamma'_{ij} e_j + \gamma'_i x, \\
[e,x] = \sum\limits_{j=1}^{n} \delta_j e_j + \delta x, & [h,x] = \sum\limits_{j=1}^{n} \sigma_j e_j + \sigma x, & [f,x] = \sum\limits_{j=1}^{n} \tau_j e_j + \tau x, \\
[x,e] = \sum\limits_{j=1}^{n} \delta'_j e_j + \delta' x, & [x,h] = \sum\limits_{j=1}^{n} \sigma'_j e_j + \sigma' x, & [x,f] = \sum\limits_{j=1}^{n} \tau'_j e_j + \tau' x, \\
[e_i,e_1] = e_{i+1}, & 1 \le i \le n-1, & [x,e_1] = -e_1, \\
[e_i,x] = ie_i, & 1 \le i \le n, &
\end{array}$$

where $1 \le i \le n$.

Now we will study the products $[R, sl_2]$ and $[sl_2, R]$.

From the following identity

$$[e,[e_1,e_1]] = 0 = [e,e_2] = \sum_{j=1}^{n} \alpha_{2j} e_j + \alpha_2 x,$$

we obtain that $\alpha_{2j} = 0$ for $1 \le j \le n$, and $\alpha_2 = 0$.

We consider the Leibniz identity:

$$\begin{aligned}
[e,[e_1,e_i]] &= [[e,e_1],e_i] - [[e,e_i],e_1] = (\sum_{j=1}^{n} \alpha_{1j} e_j + \alpha_1 x) e_i - (\sum_{j=1}^{n} \alpha_{ij} e_j + \alpha_i x) e_1 = \\
&= -\sum_{j=1}^{n-1} \alpha_{ij} e_{j+1} + \alpha_i e_1, \qquad 2 \le i \le n.
\end{aligned}$$

On the other hand, we have that $[e,[e_1,e_i]] = 0$ for $2 \le i \le n$.

Comparing the coefficients at the basic elements we obtain $\alpha_{ij} = 0$ for $1 \le j \le n-1$, and $\alpha_i = 0$ for $2 \le i \le n$, thus $[e,e_i] = \alpha_{in} e_n$ with $3 \le i \le n$.

Consider the following equalities

$$\begin{aligned}
[e,[e_i,e_1]] = [[e,e_i],e_1] - [[e,e_1],e_i] = \; & [\alpha_{in} e_n, e_1] - (\sum_{j=1}^{n} \alpha_{1j} e_j + \alpha_1 x) e_i = \\
& = 0 = [e,e_{i+1}] = \alpha_{i+1,n} e_n, \qquad 2 \le i \le n.
\end{aligned}$$

Then we have that $[e,e_i] = 0$ with $2 \le i \le n$.

$$\begin{aligned}
0 = [[e_1,e_i],e] = [e_1,[e_i,e]] + [[e_1,e],e_i] = \; & [e_1, \sum_{j=1}^{n} \alpha'_{ij} e_j + \alpha'_i x] + [\sum_{j=1}^{n} \alpha'_{1j} e_j + \alpha'_1 x, e_i] = \\
& = \alpha'_{i1} e_2 + \alpha'_i e_1,
\end{aligned}$$

it follows that $\alpha^{'}_{i1}=0$ and $\alpha^{'}_{i}=0$ for $2\leq i\leq n$.

$$\begin{aligned}[x,[e,x]]=[[x,e],x]-[[x,x],e]= \ & [\sum_{j=1}^{n}\delta^{'}_{j}e_j+\delta^{'}x,x]=\\ & =\sum_{j=1}^{n}\delta^{'}_{j}je_j.\end{aligned}$$

On the other hand, we have that $[x,[e,x]]=[x,\sum\limits_{j=1}^{n}\delta_je_j+\delta x]=-\delta_1e_1$.

Comparing the coefficients at the basic elements we obtain $\delta^{'}_{1}=-\delta_1$ and $\delta^{'}_{j}=0$ for $2\leq j\leq n$.

$$\begin{aligned}&[e_1,[e,x]]=[[e_1,e],x]-[[e_1,x],e]= && [\sum_{j=1}^{n}\alpha^{'}_{1j}e_j+\alpha^{'}_{1}x,x]-[e_1,e]=\\ &=\sum_{j=1}^{n}\alpha^{'}_{1j}je_j-(\sum_{j=1}^{n}\alpha^{'}_{1j}e_j+\alpha^{'}_{1}x)= && \sum_{j=1}^{n}\alpha^{'}_{1j}(j-1)e_j-\alpha^{'}_{1}x=\\ &=[e_1,\sum_{j=1}^{n}\delta_je_j+\delta x]=\delta_1e_2+\delta e_1.\end{aligned}$$

We obtain that $\delta=0$, $\alpha^{'}_{12}=\delta_1$ and $\alpha^{'}_{1j}=0$ for $3\leq j\leq n$.

$$\begin{aligned}&[[e,x],e_1]=[e,[x,e_1]]+[[e,e_1],x]= && -[e,e_1]+[\sum_{j=1}^{n}\alpha_{1j}e_j+\alpha_1x,x]=\\ &=-\sum_{j=1}^{n}\alpha_{1j}e_j-\alpha_1x+\sum_{j=1}^{n}\alpha_{1j}je_j= && \sum_{j=1}^{n}\alpha_{1j}(j-1)e_j-\alpha_1x=\\ &=[\sum_{j=1}^{n}\delta_je_j,e_1]=\sum_{j=1}^{n-1}\delta_je_{j+1},\end{aligned}$$

it follows that $\alpha_1=0$, $\delta_j=j\alpha_{1,j+1}$ for $1\leq j\leq n-1$.

$$\begin{aligned}&[e_1,[x,e]]=[[e_1,x],e]-[[e_1,e],x]= && [e_1,e]-[\alpha^{'}_{11}e_1+\delta_1e_2+\alpha^{'}_{1}x,x]=\\ &=\alpha^{'}_{11}e_1+\delta_1e_2+\alpha^{'}_{1}x-\alpha^{'}_{11}e_1-2\delta_1e_2= && \alpha^{'}_{1}x-\delta_1e_2=[e_1,-\delta_1e_1+\delta^{'}x]=-\delta_1e_2+\delta^{'}e_1,\end{aligned}$$

Thus we have that $\delta^{'}=\alpha^{'}_{1}=0$.

$$\begin{aligned}&[[x,e],e_1]=[x,[e,e_1]]+[[x,e_1],e]= && [x,\sum_{j=1}^{n}\alpha_{1j}e_j]-[e_1,e]=-\alpha_{11}e_1-\alpha^{'}_{11}e_1-\delta_1e_2=\\ &=-(\alpha_{11}+\alpha^{'}_{11})e_1-\delta_1e_2= && -[\delta_1e_1,e_1]=-\delta_1e_2\end{aligned}$$

it follows that $\alpha^{'}_{11}=-\alpha_{11}$.

$$\begin{aligned}&[e_i,[e,x]]=[[e_i,e],x]-[[e_i,x],e]= && [\sum_{j=2}^{n}\alpha^{'}_{ij}e_j,x]-i[e_i,e]=\\ &=\sum_{j=2}^{n}\alpha^{'}_{ij}je_j-i\sum_{j=2}^{n}\alpha^{'}_{ij}e_j= && \sum_{j=2}^{n}\alpha^{'}_{ij}(j-i)e_j=\\ &=[e_i,\sum_{j=1}^{n}\delta_je_j] && =\delta_1e_{i+1},\end{aligned}$$

Thus $\alpha^{'}_{i,i+1}=\delta_1$ and $\alpha^{'}_{ij}=0$ for $j\neq\{i,i+1\}$, $2\leq i\leq n-1$.

$$\begin{aligned}&[e_n,[e,x]]=[[e_n,e],x]-[[e_n,x],e]= && [\sum_{j=2}^{n}\alpha^{'}_{nj}e_j,x]-n[e_n,e]=\\ &=\sum_{j=2}^{n}\alpha^{'}_{nj}je_j-n\sum_{j=2}^{n}\alpha^{'}_{nj}e_j= && \sum_{j=2}^{n}\alpha^{'}_{nj}(j-n)e_j=\\ & && =[e_n,\sum_{j=1}^{n}\delta_je_j]=0,\end{aligned}$$

We get $\alpha'_{nj}=0$ for $2\leq j\leq n-1$.

$$\begin{aligned}
&[e_i,[e_1,e]]=[[e_i,e_1],e]-[[e_i,e],e_1]= && [e_{i+1},e]-[\alpha'_{ii}e_i+\delta_1e_{i+1},e_1]=\\
&=\alpha'_{i+1,i+1}e_{i+1}+\delta_1e_{i+2}-\alpha'_{ii}e_{i+1}-\delta_1e_{i+2}= && (\alpha'_{i+1,i+1}-\alpha'_{ii})e_{i+1}=\\
&=[e_i,-\alpha_{11}e_1+\delta_1e_2] && =-\alpha_{11}e_{i+1},
\end{aligned}$$

it follows that $\alpha'_{i+1,i+1}-\alpha'_{ii}=-\alpha_{11}$ for $1\leq i\leq n-1$, we have that $\alpha'_{ii}=-i\alpha_{11}$, for $1\leq i\leq n$.

Similarly, applying the Leibniz identity to the triples $\{h,e_1,e_i\}$; $\{x,h,x\}$; $\{e_i,h,x\}$; $\{f,e_1,e_i\}$; $\{x,f,x\}$ and $\{e_i,f,x\}$ for $1\leq i\leq n$, we get the following table of multiplications:

$$\begin{array}{lll}
[e,h]=2e, & [h,f]=2f, & [e,f]=h,\\
[h,e]=-2e, & [f,h]=-2f, & [f,e]=-h,\\
[e,e_1]=\sum\limits_{j=1}^{n}\alpha_{1j}e_j, & [h,e_1]=\sum\limits_{j=1}^{n}\beta_{1j}e_j, & [f,e_1]=\sum\limits_{j=1}^{n}\gamma_{1j}e_j,\\
[e_i,e]=-i\alpha_{11}e_i+\alpha_{12}e_{i+1}, & [e_i,h]=-i\beta_{11}e_i+\beta_{12}e_{i+1}, & [e_i,f]=-i\gamma_{11}e_i+\gamma_{12}e_{i+1},\\
 & & 1\leq i\leq n-1,\\
[e_n,e]=\alpha'_{nn}e_n, & [e_n,h]=\beta'_{nn}e_n, & [e_n,f]=\gamma'_{nn}e_n,\\
[e,x]=\sum\limits_{j=1}^{n-1}j\alpha_{1,j+1}e_j+\delta_ne_n, & [h,x]=\sum\limits_{j=1}^{n-1}j\beta_{1,j+1}e_j+\sigma_ne_n, & [f,x]=\sum\limits_{j=1}^{n-1}j\gamma_{1,j+1}e_j+\tau_ne_n,\\
[x,e]=-\alpha_{12}e_1, & [x,h]=-\beta_{12}e_1, & [x,f]=-\gamma_{12}e_1,\\
[e_i,e_1]=e_{i+1}, & 1\leq i\leq n-1, & [x,e_1]=-e_1,\\
[e_i,x]=ie_i, & 1\leq i\leq n. &
\end{array}$$

Using the above obtained equalities and the following

$$\begin{aligned}
&[e_n,[e,h]]=[[e_n,e],h]-[[e_n,h],e]= && [\alpha'_{nn}e_n,h]-[\beta'_{nn}e_n,e]=\\
&=\alpha'_{nn}\beta'_{nn}e_n-\beta'_{nn}\alpha'_{nn}e_n=0= && [e_n,2e]=2[e_n,e]=2\alpha'_{nn}e_n,
\end{aligned}$$

it follows that $\alpha'_{nn}=0$, we have that $0=\alpha'_{nn}=-n\alpha_{11}$, so $\alpha'_{ii}=-i\alpha_{11}=0$, for $1\leq i\leq n$.

Similarly, applying the Leibniz identity to the triples $\{e_n,f,h\}$ and $\{e_n,e,f\}$ we have that $\beta_{ii}=\gamma_{ii}=0$ for $1\leq i\leq n$.

Now we consider the equalities

$$\begin{aligned}
&[x,[e,h]]=[[x,e],h]-[[x,h],e]= && [-\alpha_{12}e_1,h]-[-\beta_{12}e_1,e]=\\
&=-\alpha_{12}\beta_{12}e_2+\beta_{12}\alpha_{12}e_2=0= && [x,2e]=-2\alpha_{12}e_1,
\end{aligned}$$

and we have that $\alpha_{12}=0$, also $\beta_{12}=\gamma_{12}=0$.

$$\begin{aligned}
&[[e,h],e_1]=[e,[h,e_1]]+[[e,e_1],h]= && [e,\sum_{j=3}^{n}\beta_{1j}e_j]+[\sum_{j=3}^{n}\alpha_{1j}e_j,h]=\\
&=0=[2e,e_1]=2\sum_{j=3}^{n}\alpha_{1j}e_j, &&
\end{aligned}$$

it follows that $\alpha_{1j}=0$ for $3\leq j\leq n$, also $\beta_{1j}=0$ and $\gamma_{1j}=0$ for $3\leq j\leq n$.

$$\begin{aligned}
&[[e,h],x]=[e,[h,x]]+[[e,x],h]= && [e,\sigma_ne_n]+[\delta_ne_n,h]=\\
&=0=[2e,x]=2\delta_ne_n. &&
\end{aligned}$$

Hence $\delta_n=0$, also $\sigma_n=\tau_n=0$.

Consequently, we obtain the following table of multiplication:

$$\begin{array}{lll}
[e,h]=2e, & [h,f]=2f, & [e,f]=h, \\
[h,e]=-2e, & [f,h]=-2f, & [f,e]=-h, \\
[e_i,e_1]=e_{i+1}, & 1\leq i\leq n-1, & [x,e_1]=-e_1, \\
[e_i,x]=ie_i, & 1\leq i\leq n. &
\end{array}$$

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