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Ma'mun Academy of Khorezm

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Beruniy's Theory of Shadows
(Gnomonics)

6nd Edition

T A S H K E N T – 2 0 20

Considering the role of one of the most ancient scientific establishments -Ma'mun Academy of Khorezm - in the history of world civilization and culture, at the 32^{nd} session of the UNESCO conference on September 29-October 17 of 2003, it was decided that UNESCO would take part in the celebration of the 1000 year anniversary of the Ma'mun academy. In order to scrutinize thoroughly the rich scientific heritage of Ma'mun Academy of Khorezm which made a great contribution to the development of world science and culture, and to develop a feeling of deep respect for the historical past of our country in the hearts of the young generation, on October 9, 2004 the Cabinet of Ministers of the republic of Uzbekistan issued a decree about the celebration of the 1000 year anniversary of Ma'mun Academy of Khorezm and confirmed the structure of the organizing committee and the relevant events.

This brochure is recommended for publishing by the scientific-methodical council of the Urganch State University.

Preface

The great encyclopaedist of the Middle Ages Abu Rayhon Muhammad ibn Ahmad al - Beruniy, whose name is associated with the Ma'mun Academy of Khorezm, was born on the 3rd of Zulhidjja month in 362 hidjra year (September 4, in 973) in the capital of South Khorezm - the Kat (near Beruniy city).

He had a great capability to science from his very early years. He was taught by the outstanding scholar Abu Nasr Mansur ibn-Irok, who had been famous with the pseudonym "Ptolemy" at that time. Abu Nasr ibn-Irok devoted 12 of his works in the field of astrology, geometry and mathematics to Abu Rayhon Beruniy, which meant respect and admitted his disciple's erudition.

In 995 the Amir of Gurgandj Ma'mun the 1st ibn-Muhammad Siavushiy occupied the Kat, which was the last fortress of Afrighians' dynasty, and declared himself the king of United Khorezm. Due to restless situation in Khorezm Beruniy had to leave home land at the age of 22. He lived in towns like Gurgan and Rai in Iran, where he became acquainted with the famous scientist Abu Mahmud Khudjandiy, as well as he established relations with ibn-Sino through correspondence and exchanged opinions with him regarding some scientific problems.

In 998 after the death of Ma'mun the 1st Ali Ma'mun ascended the throne and the political situation became more stable in Khorezm. Ali ibn-Ma'mun was a man of science, culture and education. He charged the supervisor of Beruniy Abu Nasr ibn-Irok to gather the scholars in the palace and to create conditions for scientific discussions. Approximately at the end of 1003 and at the beginning of 1004 according to ibn-Irok's invitation Beruniy returned to Gurgandj and his prosperous scientific period began. In this period a lot of the greatest scientist of the time, such as famous scientist in medicine - Ali Abu ibn-Sino, Abu Sakhl Masikhiy, Al Khodjandiy, Abdumalik as-Saolibiy and others gathered in the Ma'mun palace. Thus, starting in 1004, a scientific establishment named "Dorul Hikma va Maorif" (in some sources "Majlisi ulamo") started operating in Gurgandj. Abu Rayhon Beruniy was appointed the manager of the Academy.

In this scientific establishment, just like in the Academy of Plato in Athens and in "Baytul Hikma" academy in Bahdad, research work in many spheres of science was carried out, a lot of books were collected, translation efforts were carried out; the works of Indian, Greek and Arab scientists were studied, the fundamental works of Al-Khorazmiy and Al-Fargoniy were used in various research works. The historical scientists of the XVII-XX centuries closely studied the establishment and from the analysis of the activities of this establishment it was concluded that it was the academy of those times. It was named the Ma'mun Academy of Khorezm.

The manager of the Ma'mun Academy of Khorezm Abu Rayhon Beruniy authored about 152 scientific works, but only 30 of them have been passed to present generation. Beruniy's scientific activity was polyhedral with mathematics, physics, mineralogy, ethnography and history remaining his main focus. His works consisting of 11 books "The Konuniy Masudiy", "Geodesy", "Mineralogy", "The Monuments left by the Ancient People" (devoted to ethnography) and "Hindiston" (India) have been used as a manual for many centuries and even presently scientists are using them in their investigations.

Beruniy lived in a socio-politically complicated period, which was related to the occupation of the town Kat, where Beruniy was born and lived, the removal of the capital to Gurgandj, roving, destabilization, arrogance and corruption in the society. These feelings can be seen in the following Beruniy's sentences and aphorisms:

"Though some men possess less scientific knowledge, they behave themselves arrogantly, and even dare to insult others. Nevertheless, the difference between them was only the wealth and poorness turning the pride into guilt" (Al-Beruniy, Osan al-Bokiya).

Richness may be lost but education will remain with you forever.

Whoever hurts a harmless scholar and enjoys it, he will be punished by the Allah.

If the branches of the evil are many, its source is bribery and ignorance.

Our great ancestor al-Beruniy died in 362 hidjra year, i.e. on the December 13, in 1048 at the age of 75 in the town of Ghazna. About the last days of the scholar the following words were mentioned in the book "Nomaiy Donishvaron", which was published in 1878 in Teheran: "Beruniy had a serous illness, he was living his last days. When he regained consciousness for a moment, he could see his friend Abdulhasan Valvolidjiy. Beruniy asked his friend Abdulhasan to comment on the new opinion about the heritage. Abdulhasan replied that it wasn't appropriate moment for it. Then, looking at his friend, Beruniy said, "Oh, my dear friend, every person is sure to die, but my mind is making me now to understand the importance of the problem, which you told me some years ago. So it is better to die knowing than to die not knowing," - answered Beruniy. His friend Abdulhasan began to comment on the things, which he had asked him to explain. In some moments Beruniy fell asleep forever. And this was his last talk about the science." How a good death! It was a death worthy of great person, it was a death of a person who had spent his life profoundly, it was a death of a scholar who had been satisfied with his activity.

This brochure is a pearl of Beruniy's activity, which concerns geometry. In this booklette together with introducing some of the Beruniy's mathematical investigations, discussions, some commentaries are also given. Unlike his forerunners Beruniy had a capability of thinking logically, reflecting the importance of chosen problem correctly and finding out a simple form of expression for his ideas. Beruniy's above-mentioned essential qualities are seen vividly in his works which are remained in the history as determining of the distance from the Earth to the Moon and the Sun, where he showed the attempts, vagueness and confusions of his forerunners, discussions about the unit of measurement, gave vivid explanations to them and demonstrated the character of a leader in the field of natural sciences.

In order to fully cover the historical roots of Ma'mun Academy of Khorezm and the life and work of al-Beruniy, we must first briefly review the life and work of Muhammad Al-Khorazmiy - the head of the academy "Baytul Hikma", which had operated in Bahdad approximately 200 years before the Ma'mun academy, and which played a vital role in the establishment of the Ma'mun academy.

Muhammad al-Khorazmiy and academy "Baytul Hikma"

Abu Ja'far Muhammad ibn Muso al-Khorazmiy was born in 783 in one of the greatest cultural centers of Khorezm - in the city of Khiva. He acquired his elementary education in the madrasahs of Khiva, Kot and Gurganch (Urganch) and during his youth studied mathematics, astronomy, and other natural sciences; scrutinized books written in Arabic, Persian, Syriac, Hindu and Greek. Due to his sharp perception and rare talent, he developed as a leading scientist while living in Khorezm; his name soon spread to the whole of the East.

Halifa Horun ar-Rashid's second son Abul Abbos Abdulloh al-Ma'mun was appointed the Naib (ruler) of Halifat's eastern lands by his father in 809. The capital of these lands was the city of Marv in Khurasan. Since his youth, al-Ma'mun was thirsty for knowledge and very intellectual. Besides dealing with state matters in Marv, he performed many activities in the fields of science and education; he gathered young scientists eager to gain more knowledge around him. Great scholars such as Muhammad al-Khorazmiy, Yahyo ibn Abu-Mansur, Holid al-Marvarrudiy, Sanad ibn-Ali, Abbos al-Javhariy, Ahyo ibn-Aksam, Kozi Hammod, Ahmad al-Marvaziy were among these scientists, and their works served as the foundation of the world science later on. After the death of his father al-Ma'mun gained the throne of the halifat and hi 813 he brought a group of scientists with him to Bahdad. Al-Ma'mun joined the scientists who came from Marv with those of Bahdad to form a scientific establishment called "Bayt Hikma". At first he appointed Yahyo ibn Abu Mansur as the head of this scientific team. After the death of Yahyo in 829, Muhammad al-Khorazmiy was appointed the head of "Bayt Hikma" and headed the establishment till the end of his life. Al-Khorazmiy conducted scientific research in collaboration with the scholars from Middle Asia such as Ahmad Fargoniy, Abuvafo Buzjoniy, Isoq Ibodiy, Abu Ja'far Hurosoniy, Abu Fatih Isfihoniy, al-Marvaziy and others.

The scientists from Middle Asia provided tremendous support for the activities of "Bayt Hikma". According to chech historical scholar Yu.Rushka, the majority of the scientists in this scientific establishment were from Khorezm, Ferghana, Shosh and Huroson. And the Khorezmians formed almost two-thirds of this "majority". It's interesting to note, that a famous Arab geographer ibn-Havkal, when describing the lands of Khorezm in his work "Roads and Counties" written in the beginning of the X century, mentions the thirst for knowledge and the sharpness of the people of Khorezm. "At the moment in Bahdad there are no qozi, faqih or scholars who don't have a disciple from Khorezm", - writes ibn-Havkal.

Headed by al-Khorazmiy, the scientists of "Bayt Hikma" performed a great amount of translation work, translated the works of scholars from Syria, Iran, Greece, India into Arabic; watched the celestial bodies, organized scientific expeditions to different parts of the halifat.

Al-Khorazmiy is the author of number of scientific works, which brought him fame at his time. Some of his books "Al-Jabr val-Mukobala" (Equation and Contrary Calculation), "Hisob al-Hind" (Indian Calculation), "Zidjy Khorazmiy" (Khorazmiy's Table), "Kitob Surat al-Arz" (About the Picture of the Earth), "Kitob al-Amal Astrolabe" (A Book about the Ways of Working with Astrolabe), "Kitob at-Tarich"(A Book of History), "Kuiash soaty hakida kitob" (A Book about the Sun Watch) have come up to our days. Each of the Khorazmiy's works became new discoveries and brought him fame.

Particularly, two works of al-Khorazmiy devoted to the arithmetics, algebra - "Equation and Contrary Calculation" and "Indian Calculation" played an important role in developing mathematical science not only in the East but in the West as well.

At that period Indian's book "Big Sindhind" was an essential manual in the East and it had been translated into the Arabic language by Muhammad ibn Ibrohim al-Fazoriy". But some parts of the book were difficult to understand and there were many mistakes, lacks in this book. The Caliph al-Ma'mun charged al-Khorazmiy to revise and to make some additions to the book. This book, revised by the scholar, was famous for its name "Short or Small Sindhind" in the East. After having been revised based on al-Khorazmiy's experiences, this work turned into a new scientific work.

Al-Khorazmiy begins the book "Indian Calculation" with the words: "Let's praise the merciful and kind Allah with beyond praises, let's express our nobility, praise to the skies, let's beg alms Him lead us to the justice and right way, when I saw the Indians to make any numbers by placing and composing 9 letters, if the Allah blesses, in order to make easy for the learners I decided to show what can be composed from those 9 letters. If the Indians want just this case and the meanings of the 9 letters are the same in their understanding and in my comprehension, let the Allah direct me to it. But if they make in another way, in this case they are sure to find the right way easily in my composition and it will be visible clearly for both the learner and teacher".

It is obvious, when al-Khorazmiy created his mathematics he didn't take the whole Indians' mathematics but took only the idea of making any numbers with the help of decimal numeration system 1,2,3,4,5,6,7,8,9 figures - letters. Before imagining these circumstance let's return back to the VIII-XI AD in our thoughts: it was the period of people's marking the figures, using various ways and systems of additions, multiplications and that's why there was a great need for a new mathematical manual at that time for, on the one hand, much muddles in the Roman's 12,20,60 numeration system and the Indian's decimal numeration system, absence of definite rules, on the other hand, the development of trade relations between the countries. The book "Small Sindhind" was an opportune manual of that time, because based on the decimal numeration system figures and addition, subtraction, multiplication, division of them and working with them were explained in a very easy way. He decided to add the figure "0" to the decimal numeration system in order to complete the decimal numeration system.

Soon the book "Small Sindhind" became famous in the world. According to the outstanding historian-scholar Jamoliddin Kiftiy's opinion, this work was the rarest manual for the scholars up to the XIII AD. Indeed this work opened new epoch in the domains of astronomy and mathematics both in Europe and in the East. In the XII AD the European scholar Adelard translated this work into Latin. The translation of this manuscript, belonging to the XIV AD, is kept in the library of the Cambridge University in Great Britain.

Any successive calculating process of the mathematical concept - algorithm is also connected with the name of al-Khorazmiy. In 1849 European orientalist-scholar J.Reino defined that the word "algorithm" had derived from the name of al-Khorazmiy, because the scholar's latinized name had been changed into algorithm, algorithmus in the West. Nowadays the concept algorithm is used widely not only in mathematics but also in cybernetics, economics.

The scholar's book "Equation and Contrary Calculation" is the fundamental of the modern algebra. Indeed he founded algebraical science for in his above mentioned work the word "al-Jabr" was used for the first time and it took the name algebra which was a new branch of mathematics. In the XII AD the European scientists Herardo and Robert Chesterskiy translated the work into Latin and the Latin, Arabic translations have come up to us.

Al-Khorazmiy spent the last more than 40 years of his life in Bahdad and devoted them to the scientific, creative activity in Bait ul-Hikma. He hadn't the opportunity to call at his native land, Khorazm, during the last half a century of his lifetime. According to some sources, al-Khorazmiy died at the result of sudden heart attack in the observatory, where he had supervised in 850. He was 67 years old and his scientific activity was flourishing. He was buried in the Shamasiya cemetery near the observatory in Bahdad.

Al-Khorazmiy's death was a great loss for the scientific world. Almost all the Bahdad's people and the outstanding scholars of the East took part in the great scholar's funeral procession. It is said that even the nature also mourned over the sudden death of the scholar. Just, in those solemn moments some unexpected events happened in the Sky of Bahdad. There happened earthquake, the Sky was covered with dark clouds. There happened strong thunderstorms and lightning in the Sky which shook the Earth. Unexpected downpour began and continued for three days. The downpour overflowed the Dadjla and Tigro rivers. The leaders of religion understood those events of that time as the Allah's grief over the sudden death of the great scientist, al-Khorazmiy.

In 1983 by the Decree of UNESCO the 1200th anniversary of the great son of the Uzbek people, the outstanding encyclopaedic scholar, al-Khorazmiy, was celebrated widely all over the world.

The Theory of Shadows

Measurements of the celestial bodies (the Earth, the Moon, the Sky), measuring the distance from the point where we stand up to them attracted attention of the most scientists from the very ancient times. The scholars from the Ma'mun Academy of Khorezm were interested in this problem as well and first of all, its head Abu Rayhon Beruniy made a great contribution to this field.

In order to determine the measures of the Earth, the Moon and the Sun, and to determine the distances from the Earth to the Moon and to the Sun, Beruniy created a theory of shades, which was perfect from the mathematical point of view. The essence of this theory is that, if we put a circle with the radius equal to d towards the Sun in some t distance from the place where we stand, the circle's full shade (in this point the circle covers the Sun completely) and partial shade (in this point the circle covers some parts of the Sun) fall onto the Earth. Based on the measurements of these shades Beruniy created a method of calculating the distance from the Earth to the Sun as well as the measures (size) of the Sun.

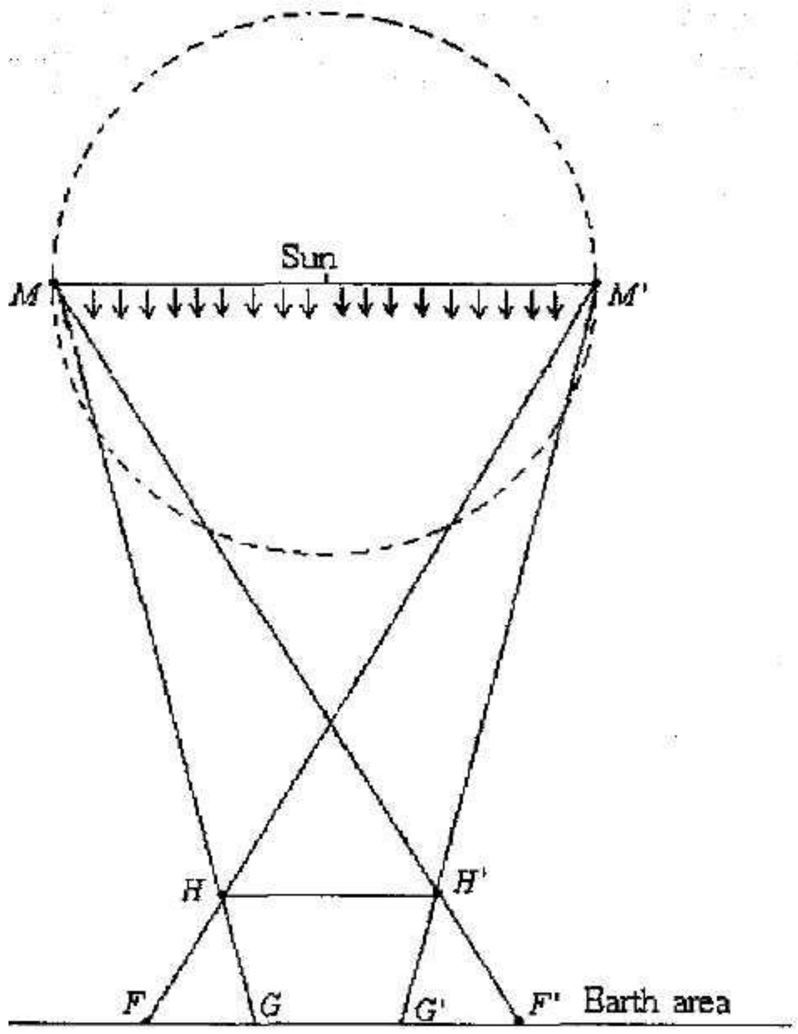

Figure 1

Here *MM'* is the diameter of the Sun, *HH'* is the diameter of the circle (gnomon), *GG'* - the area of the full shade of gnomon, *FG* and *F'G'* - the area of the partial shade of *HH'*.

In this brochure we shall see and analyse the methods, created by Beruniy for measuring the radius of the Earth and the distance from the point where we stand to the object, which is far from us; also we pay attention to using them in modern practice, in making mathematical manuals. We'll also cite Beruniy's own sentences, some passages from his books. While reading them, the reader will for sure come to conclusion that Beruniy was and remains a great mathematician from the point of view of modern science as well.

Measuring the Distance on the Ground and the Height of a Mountain

If we are to measure the height AB of some standing object (for ex. the height of a minaret) we go to a point *C* which is at some distance from the object (Figure 2), measure the angle *a* using a leveling instrument and from the equation

$tg\alpha = \frac{AB}{BC}$ or $AB = BC \cdot tg\alpha$ we can determine the length of AB easily.

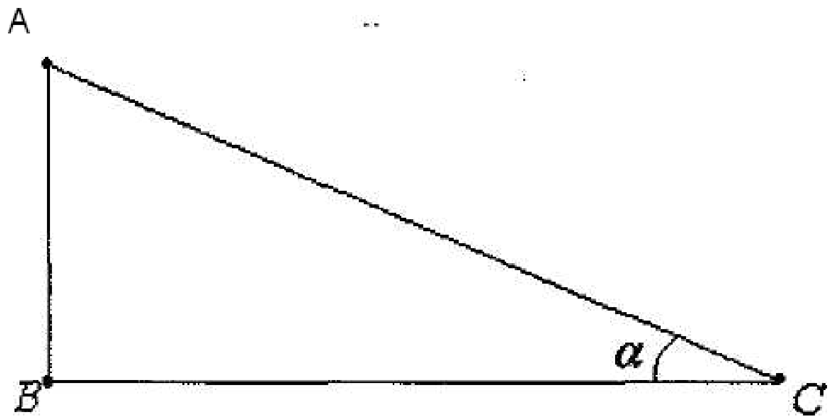

Figure 2

If we can't get to the basis of the object *B,* for example, if we are to measure the height of an object on the other side of a river or the height of the plateau, the task will become more complicated. Al-Beruniy wrote about such cases in details in his work "Gnomonics" and gave the solutions to such problems by using the Indian mathematician and astronomer Brahmagupta's method from *"Brahmassiddhta"* (one of the greatest books of Brahmagupta). According to his opinion in order to measure the height of the objects with inaccessible basis, we should choose a flat place at some distance from the object (Figure 3).

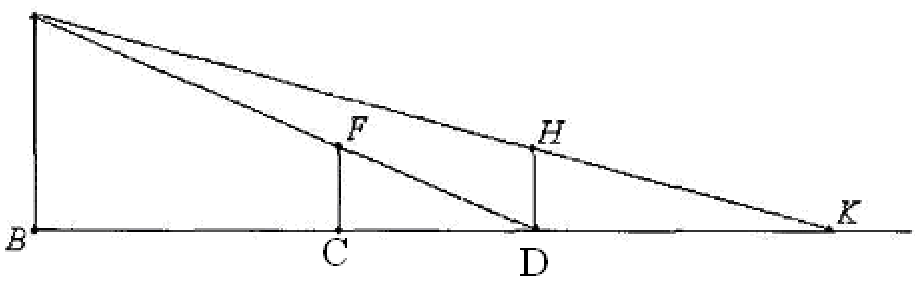

Figure 3

We select a point *C* on a flat site, we place a gnomon *CF* vertically and find out its full shade *CD.* In order to find out point *D,* which defines the full shade of *CF* gnomon Beruniy stated the following: "… then one should reach back from the point C to such a place, where from level's diopter *F* and *A* should be seen in the same landmark ... as the point *D* is on the ground you can either lie on the ground or dig a pit of depth equal to your stature, descend into the pit and look through the diopter" (Al-Beruniy Mathematical and Astronomical Treatise, "Fan", 1987, p. 244)

After having determined the point *D,* we raise at point *D* another gnomon *DH,* which is equal to *CF* gnomon and find out its shade *DK* in the same way as the with previous one.

From the similarities $\Delta ABD \sim \Delta CDF$, $\Delta ABK \sim \Delta DKH$ come out the equations

$$\frac{AB}{BD} = \frac{CF}{CD}, \qquad \frac{AB}{BK} = \frac{HD}{DK},$$

and using these equations it is not difficult to find out

$$BD = \frac{CD \cdot DK}{DK - CD}, \quad AB = \frac{CF \cdot DK}{DK - CD}$$

Beruniy created a simple and very easy way of measuring the distances on the surface of the Earth in his book "Geodesy" (Al-Beruniy, Geodesy, "Fan", 1982, p. 167).

For this he took *CDEF* square with the sides equal to 1, knocking nails into the points *C* and *D,* established long diopterical level to the point *F* (Figure 4).

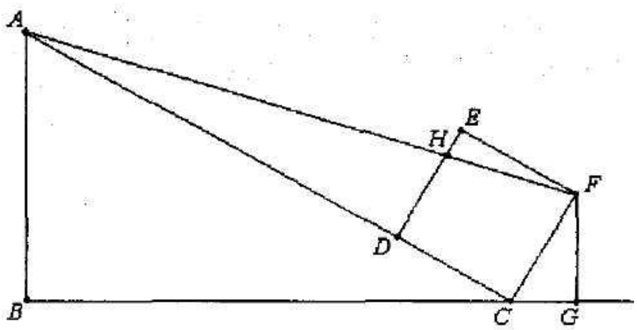

Figure 4

It is necessary to establish the square to the point *C* in such a way so that the points *A,D,C* lay on one straight line. Then we drop a stone from the point *F* (with the Beruniy's words) and draw perpendicular *FG*.

From

$$\Delta ACF \sim \Delta EFH, \quad \Delta ABC \sim \Delta CFG$$

we shall obtain the equalities

$$AC = \frac{1}{EH}, \quad AB = \frac{CG}{EH} \tag{1}$$

Measuring of the Earth

The **1st** attempt to measure the diameter of the Globe is connected with the name of Eratosfenes (276 - 196 BC). He determined the measurements of the Globe according to the position of the Sun in Aswan and Alexandria. According to his determination, when in Aswan the Sun is at Zenith, in Alexandria it reaches $7{,}2^0$ in relation to Zenith and from this, having specified, that on the globe the arch connecting Aswan and Alexandria, reaches $7{,}2^0$, determines its conformity to 1/50 part of the large circumference of the Globe.

So he multiplied the distance between Aswan and Alexandria by 50 and calculated the length of the Earth's large circumference. Ptolemy (II CE) also tried to calculate the measurements of the Globe and he expressed his ideas about the parameters of the Earth in his book "Geography". But the scholars of the ancient times used "Stadiy" as a unit of measurement and as time passed by, especially during the academy "Bait ul-Hikma's" period (IX century), it turned out that due to the vagueness and contradictions among the units of measurement, there were many mistakes in the values of the Earth's measurements.

Therefore, the chalif al-Ma'mun ibn ar-Rashid charged the scholars of the academy "Bait ul-Hikma" with the task of determining the real measurements of the Globe. Observations were carried out in the Sindjor desert near Mosul and mostly the Middle Asian scholars took part in this investigation work. Under the leadership of our countryman al-Khorazmiy, the scholars of "Bait ul-Hikma" fulfilled the task successfully. They determined that the radius of the Earth was equal to 3247 *mil* = 129864996 *gaz* or 6406 *km*[1]*)* in fact, the actual radius of the Earth at the equator is equal to R = 6378 *km* and at the Polar circle is equal to 6357 *km.*

Describing in details the attempts of his forerunners in measuring the size of the Earth in his books "Geodesy", "Konuniy Masudiy" Abu Rayhon Beruniy offered another new method. Beruniy wrote: "Only Greek and Indian versions of measurement of the Earth came up to us. Greek's and Indian's units of measurement were different, for example one *mil,* which Indians used to measure the circumference of the Earth was equal to quantity between one and eight our *miles* and the various measurements confused their thoughts for various scholars had

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[1] The modern scientist established that 1 *mil* = 4000 *gaz* = 1973,2 *meters* (see, Hints "Muslim Measurements")

different results. In each of their five "Siddihonta's" the value of the Earth circumference was different. But Greeks measured the circle of the Earth by one quantity, which was called *"stadiya*". According to Galen, Eratosthenes carried out observations in Aswan and Alexandria, which are situated on the same meridian.

Whenever the words in Galen's book "The Book of Provements" are combined with the words from Ptolemy's book "Entering the Art of Sphere" again the quantity will be different. Therefore, Ma'mun ibn ar-Rashid charged the leaders of the science, who carried out the investigation in the Sindjor desert of Mosul to pay attention to such contradictions.

If any man moves along a straight line on the Earth plane, he will move along the big circle of the Earth. But it is difficult to pass far distance along the straight line. That's why the scientists of the Ma'mun Academy of Khorezm took the pole of the Earth as a reference point (the pole-star seems to be meant here, author's note). Being careful, they determined that one part of circle in 360^0 was equal to $56\frac{2}{3}$*miles.*

I myself was eager to measure the Earth and I chose a large plane land in Djurdjon. But because of the inconvenient condition of the desert, the absence of the people, who could help me out, I found a high mountain with a smooth surface in the lands of India and used a different way of measuring it. From the top of the mountain I found the horizont of the Sky and Earth (Figure 5) and calculated its angle, which was equal to $0^0 34'$, measured the top mountain in two places and it was equal to 652 *gaz* and a half of the one-tenth of the *gaz* add.

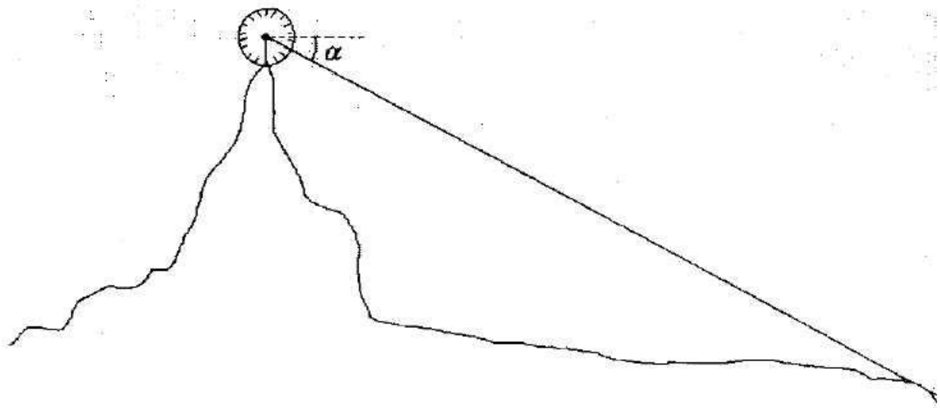

Figure 5

Let the line *CE* which is perpendicular to the sphere of the Earth be the height of the mountain (Figure 6).

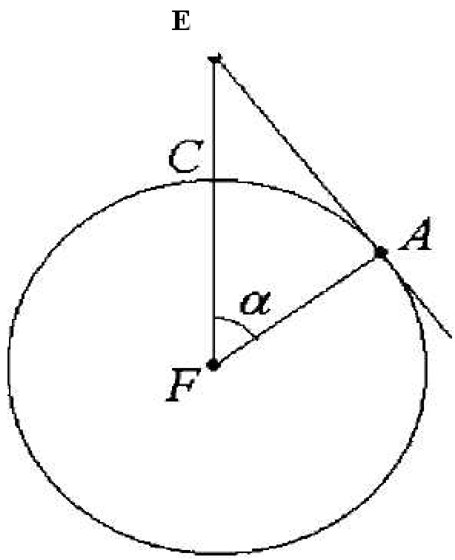

Figure 6

The centre of the Earth is *F*, the line originating on the top of the mountain and going towards the horizont is *AE*, and we shall draw *AF* perpendicular to the horizont line. Consequently, we get triangle *AFE*.

Its angle *A* is a right angle and all other angles are known. Because the angle *AEF* is the supplement angle of the horizon slope angle, that is, $90^0 - \alpha = 89^0 26'$..." (al-Beruniy, Konuniy Masudiy, book 5, 1973, p.p. 386 - 387).

So, according to the definition of sine, the radius of the Earth R is calculated. From ΔAEF we get $\sin\left(90^0 - \alpha\right) = \dfrac{AF}{AF + CE}$

and from this

$$AF = \frac{CE \cdot \sin\left(90^0 - \alpha\right)}{1 - \sin\left(90^0 - \alpha\right)} \quad \text{or} \quad AF = \frac{CE \cdot \cos\alpha}{1 - \cos\alpha} \qquad (2)$$

Knowing the height of the mountain *CE* and the value of $\sin 89^0 26'$ Beruniy established, that the radius of the Earth is R = 122851370 *gaz* = 3212,8 *mil* = 6340 *km*.

In his book "Geodesy" Beruniy also wrote that, during Chaliph al-Ma'mun's marching to Greece (830 - 832) he asked the mathematic scholar Abu Taiyib Sanad ibn Ali, who also was with him, to ascend a mountain which stuck out of the East side of the Sea and from its top to determine the lower angle (for accuracy, during the sunset), and that when he fulfilled the task, they calculated the radius of the Earth, using the lower angle and some additional angles (al-Beruniy, "Geodesy", "Fan", 1982, p. 166).

The Distances between the Celestial Bodies

Beruniy writes: "Let the sun's diameter be denoted as *AB*,

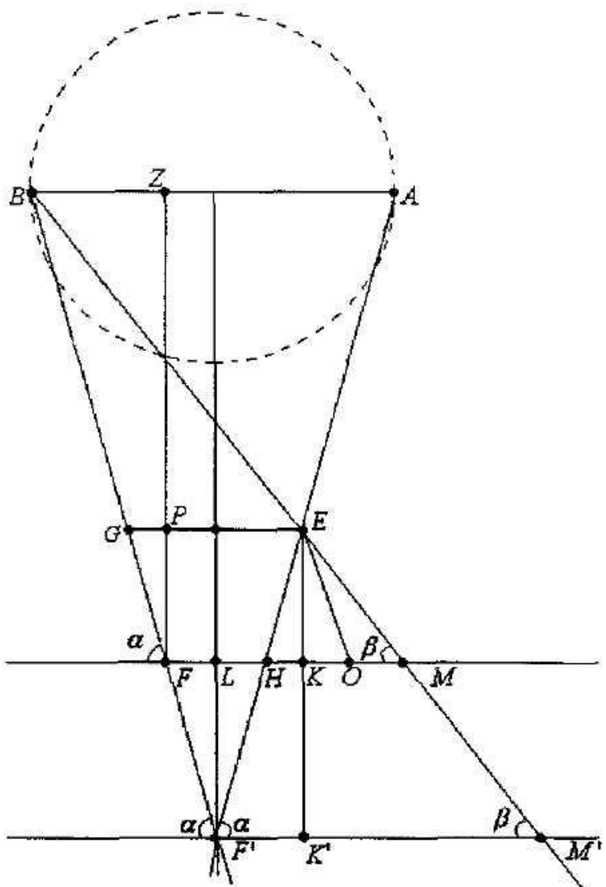

Figure 7

FM - the surface of the Earth, *EG* - gnomon object which produces a shadow, *FH* is the shadow's diameter of this object on the Earth, *L* is the centre of the shadow (Figure 7, in this drawing *FH* - full shadow, *HM* - partial shadow). If we know *FH*,*HM*,*EG* and *EK* (EK $\perp$ FM), we shall obtain the distance from the Sun to the Earth and the diameter of the Sun*).

Indeed, if we draw $EO \parallel GF$, then $OF=GE$, and *OM* is known. Its ratio to *EM* is like the ratio of *FM* to *MB*. That is, *MB* and the triangle *FMB* are known. The ratio of *FG* to *GP* is the same as the ratio of *FB* to *BZ*. That is, *BZ* is known and from that *FZ* is known (Al-Beruniy, Mathematical and Astronomical Treatise, "Fan", 1987, p. 210).

According to Beruniy's proof, $\Delta\, BFM \sim \Delta\, EOM$, from this

$$\frac{BM}{FM} = \frac{EM}{OM}, \quad \text{or } BM = \frac{EM}{OM} FM$$

$$\frac{FB}{FM} = \frac{EO}{OM} \quad \text{or} \quad FM = \frac{EO}{OM} FM$$

come out. From $\Delta FGP \sim \Delta FBZ$ follows that

$$\frac{FZ}{FB} = \frac{FP}{FG} \quad \text{or} \quad FZ = \frac{FP}{FG} FB$$

$$\frac{BZ}{FB} = \frac{GP}{FG} \quad \text{or} \quad BZ = \frac{GP}{FG} FB .$$

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* Above mentioned *FH*, *HM*, *EG* and *EK* quantities are distances, which can be measured being on the Earth

By these equalities we can determine easily the distance from the Earth to the Sun $L=FZ$ and the radius of the Sun $R = BZ + LH$:

$$L = \frac{EK \cdot FM}{FM - EG}, \qquad R = \frac{GP \cdot FM}{FM - EG} + \frac{FH}{2}, \tag{3}$$

where

$$FM = FH + HM, \qquad GP = \frac{EG - FH}{2}$$

If we mark the acute angles in the points F and M with α and β, by applying the theorem of sinuses to ΔEOM the formulas (3) can be changed into the following:

$$L = \frac{\sin\alpha \sin\beta}{\sin(\alpha - \beta)} FM, \qquad R = \frac{\cos\alpha \sin\beta}{\sin(\alpha - \beta)} FM + \frac{FH}{2} \tag{4}$$

Finally, by continuing the BF and AH straight lines we s hould Find point F' and draw a straight line $F'M' \parallel FM$. Drawing $EK' \perp F'M'$, and using the definitions of $tg\alpha$, $tg\beta$ we shall have

$$L = d\, tg\alpha \frac{tg\beta + tg\alpha}{tg\alpha - tg\beta} - d' tg\alpha$$

$$R = d \frac{tg\beta + tg\alpha}{tg\alpha - tg\beta} \tag{5}$$

formulas, here $d = \frac{EG}{2}, \quad d' = \frac{FH}{2}$

The formulas (3), (4), (5) are formulas of measuring the distance from the Earth to the celestial bodies the Moon and the

Sun and their size. Unfortunately in practice, when we use rudimentary equipment for measurement and as the Moon and the Sun are too far from us, the vales FM and EG or the angles α and β are almost equal to each other, and the denominators of the above fractions in the formula's are also almost equal to 0. Therefore, during Beruniy's period there was no possibility to use the formulas in measuring the astronomical objects. Beruniy wrote in details about the attempts in measuring the astronomical objects, about vagueness and confusion in the measurement, and although he offered theoretically simple and easy formulas for such measurements, he didn't introduce any definite figures concerning the measurements of the Moon and the Sun.

But we can use the formulas suggested by Beruniy to measure the height of unapproachable objects from the surface of the Earth, which are far from us, also to measure the distance up to them. It would be just to call these formulas the **formulas of Beruniy** and to connect his theory of gnomon (shadows) with his name.